Theoretical and Computational Acoustics
D. Lee, Y-H Pao, M.H. Schultz, and Y-C Teng (Editors)

Numerical Recognition of Seabed Parameters Using Data of Reflections

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ABSTRACT

A seabed model consisting of three, plane and parallel layers is studied. The layers are characterized by the following qualities: (1) The water column by the velocity of sound and the density, which are assumed known, (2) a homogeneous layer of sediments described by four parameters, the longitudinal wave velocity, the shear wave velocity, the density and the thickness, which are unknown, and (3) a bedrock described by three unknown parameters, the longitudinal wave velocity, the shear wave velocity and the density. All seven unknown parameters are determined through an inverse procedure. The forward propagating wave problem is solved based on a modified version of the Thomson-Haskell's matrix method. Lossless propagation is assumed. The backward propagating wave problem, i.e. the reflection problem, includes numerical studies of the reflected signals in the time- and frequency domains. The inverse procedure is evaluated by restoring the input data for the forward propagation from the information contents of the reflected waves. A good agreement between the restored data and the initially assumed data is found.

INTRODUCTION

The inverse problem of determination of seabed qualities using acoustic techniques is of current interest. The parameters to be determined are the density, velocity profiles and layer thickness. These parameters, together with the attenuation coefficients, are sufficient for a rough description of the structure of the seabed layers and form the necessary input data for the forward problem of underwater acoustic signal propagation.

Attention has been paid to inverse procedures by several investigators over the past[1–4]. A number of inverse procedures have been developed to recover some of the seabed parameters by using experimental data and numerical algorithms[3,4]. The purpose of the present work is to recover seven unknown parameters (longitudinal and shear wave velocities, density, and thickness of a sediment layer, and longitudinal and shear wave velocities and density of the substrate layer) in a three-layer seabed model. Attenuation is not taken into account, as it can be evaluated by other means[4].

In an inverse procedure, the experimental data are the only information that can be used in restoring the required parameters. The information includes, the incident time signal and several reflected time signals measured at different angles of incidence. An FFT technique is used in the procedure to obtain the frequency spectrum of the time signals. By comparing the spectrum of the incident signal and the spectra of the reflected signals, the dependence of the reflection coefficients on the frequency can easily be determined and used in the inverse procedure.

As an inverse procedure is always based upon solutions to the forward problems, numerical simulations are performed first. Starting from a modified Thomson-Haskell's matrix method, the forward problems are analyzed theoretically and simulated numerically. Through the theoretical analyses, expressions relating the incident signal, the model parameters, and the reflected signals are derived, forming basis for the establishment of the inverse procedure.

Using the numerical reflection results from the forward procedure, the model seabed qualities are reconstructed numerically. Recovered parameters are compared with the pre-assumed ones.

I. THE FORWARD PROBLEM

A forward problem concerning reflection of acoustic signals from a multi-layer model has been performed based upon the Thomson-Haskell's matrix method. Numerical simulations have been made and compared with experimental results. Good agreement has been obtained.

A. Fundamentals of wave reflection from multi-layer models

The reflection of acoustic waves from multi-layer models has been extensively studied[5−9]. A typical multi-layer model, shown in Fig.1, consists of $(n+1)$ layers. For monochromatic, plane wave reflections from such a model, the Thomson-Haskell's matrix method[5,6] forms an appropriate technique. Four waves are involved, i.e. the forward and backward propagating longitudinal waves, ϕ_l'' and ϕ_l', and the forward and backward propagating shear waves, ψ_l'' and ψ_l', propagating in the lth layer $(l = 2, ..., n)$, where ϕ and ψ are velocity potentials. Assuming a monochromatic plane wave incident on the upper interface of the multi-layer model as shown in Fig.1, and by using the boundary conditions, the total reflection coefficient can be derived as[8]:

$$R = \frac{M_{32} - Z_1 M_{33} + (M_{22} - Z_1 M_{23}) Z_{n+1}}{M_{32} - Z_1 M_{33} - (M_{22} - Z_1 M_{23}) Z_{n+1}}, \tag{1}$$

where

$$\begin{aligned} M_{22} &= A_{22} - \frac{A_{21}A_{42}}{A_{41}}, M_{23} = A_{23} - \frac{A_{21}A_{43}}{A_{41}}, \\ M_{32} &= A_{32} - \frac{A_{31}A_{42}}{A_{41}}, M_{33} = A_{33} - \frac{A_{33}A_{43}}{A_{41}}, \end{aligned} \tag{2}$$

Z_1 is the impedance at the interface between layers 1 and 2, and Z_{n+1} is the impedance at the interface between layers n and $n+1$. A_{ij} are elements of the matrix $[A]$, given by

$$[A] = [A^{(n)}] \cdots [A^{(l)}] \cdots [A^{(2)}], [A^{(l)}] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}, \tag{3}$$

where $[A^{(l)}]$ is the connecting matrix between layers l and $l-1$. The 16 elements of the matrix comprise medium parameters and signal information[8].

B. The seabed model treated in the present work

A three-layer model is assumed for the seabed. The upper layer is water, of which the longitudinal wave velocity c_3, and the density ρ_3, are known (no shear wave is supported in water column). There is only one sediment layer, of which the longitudinal and shear wave velocities c_{2p} and c_{2s}, the density ρ_2, and the thickness d, are unknown. Thus four parameters are to be determined for this layer. In the substrate below the sediment layer, the longitudinal and shear wave velocities c_{1p} and c_{1s}, and the density ρ_1, are unknown. Three parameters are to be recovered for this layer.

C. The reflection coefficient for normal incidence

A numerically generated tone-burst (shown in Fig.2a) is incident onto the upper interface of the seabed model, and the reflected signal is plotted in Fig.3a. All the analyses will be based on these two time signals and their frequency spectra.

The ratio of the frequency spectrum of the reflected signal (Fig.3b) to that of the incident signal (Fig.2b) shows some dependence of the reflection coefficient on the frequency (Fig.4). This frequency dependence can be explained by the expression for the reflection coefficient for a plane wave incident onto the water-sediment interface in the three-layer model given by[8]

$$R = \frac{R_{32} + R_{21} e^{i4\pi df\cos\theta_2/c_{2p}}}{1 + R_{32}R_{21}e^{i4\pi df\cos\theta_2/c_{2p}}}, \tag{4}$$

where R_{32} and R_{21} are plane wave reflection coefficients from layer 3 to layer 2, and from layer 2 to layer 1, respectively, given by:

$$R_{32} = \frac{\rho_2 c_{2p}/\cos\theta_2 - \rho_3 c_3/\cos\theta_3}{\rho_2 c_{2p}/\cos\theta_2 + \rho_3 c_3/\cos\theta_3}, R_{21} = \frac{\rho_1 c_{1p}/\cos\theta_1 - \rho_2 c_{2p}/\cos\theta_2}{\rho_1 c_{1p}/\cos\theta_1 + \rho_2 c_{2p}/\cos\theta_2}, \tag{5}$$

f is the frequency, and θ_3, θ_2 and θ_1 are angles with the normal to the interface for longitudinal wave propagation in the different layers. Eq.(4) is a simplified version of Eq.(1) and only good for fluid model.

But this expression can still be used for a real seabed model when the signal is normally incident on the interfaces since no shear waves are generated in this case.

II. THE INVERSE PROCEDURE

A. Basic expressions used in the inverse procedure

From Eq.(4) and Fig.4 it may be seen that the reflection coefficient for a three-layer model is a periodical function of frequency f. Based upon Eq.(4), the maximum value of the reflection coefficient can be determined as:

$$R_{max} = \frac{R_{32} + R_{21}}{1 + R_{32}R_{21}} = \frac{\rho_1 c_{1p}/\cos\theta_1 - \rho_3 c_3/\cos\theta_3}{\rho_1 c_{1p}/\cos\theta_1 + \rho_3 c_3/\cos\theta_3}, \tag{6}$$

and the frequency for the periodic variation of the reflection coefficient is given by:

$$\Delta f = c_{2p}/2d\cos\theta_2. \tag{7}$$

R_{max} and Δf can be easily measured from the total reflection coefficient shown in Fig.4. Eqs.(6-7) can then be used in the inverse procedure.

B. Further considerations

In Fig.3a, the first echo is the reflection from the interface between water and sediment, the second is from the interface between sediment and substrate, while the third and later echoes are multi-reflections from the two interfaces. The amplitudes for different reflected echoes change, but they are all replicas of the incident signal plus a phase shift. The spectrum for each of reflected echoes will also be a replica of that for the incident signal. Reflection coefficients for different reflected echoes versus the angle of incidence are plotted in Fig.5.

Numerical simulations show that the angle α_1, corresponding to the first peak of the curve for the second echo is the critical angle for the longitudinal wave in the substrate layer, and the angle α_2, at which the curve for the second echo reaches zero, is the critical angle for the longitudinal wave in the sediment layer, i.e.

$$c_{1p} sin\alpha_1 = c_3, \tag{8}$$

$$c_{2p} sin\alpha_2 = c_3. \tag{9}$$

The reflection coefficient for the first reflected echo is equal to

$$R_1 = R_{32} = \frac{\rho_2 c_{2p}/cos\theta_2 - \rho_3 c_3/cos\theta_3}{\rho_2 c_{2p}/cos\theta_2 + \rho_3 c_3/cos\theta_3}. \tag{10}$$

The recovery of shear wave velocities is usually considered to be very difficult. By comparing the two curves in Fig.6, however, it may be seen that when shear waves are present in the sediment layer, small peaks are added on the main curve of the reflection coefficient. The frequency δf, based on the interval between two small peaks is a measure of the shear wave velocity, i.e.

$$\delta f = c_{2s}/2d cos\gamma_2, \tag{11}$$

where c_{2s} is the shear wave velocity and γ_2 is the angle of shear wave propagation in the sediment layer.

For an elastic model where shear waves can be generated, the maximum value of the total reflection coefficient given in Eq.(6) will be lower due to the interaction between different waves (Fig.7). This maximum reflection coefficient R_x, is now given by[8]

$$R_x = \frac{\rho_1 c_{1p} cos^2 2\gamma_1/cos\theta_1 + \rho_1 c_{1s} sin^2 2\gamma_1/cos\gamma_1 - \rho_3 c_3/cos\theta_3}{\rho_1 c_{1p} cos^2 2\gamma_1/cos\theta_1 + \rho_1 c_{1s} sin^2 2\gamma_1/cos\gamma_1 + \rho_3 c_3/cos\theta_3}, \tag{12}$$

where γ_1 is the angle of shear wave propagation in the substrate layer.

C. The full algorithm of the inverse procedure

For seven unknowns, seven expressions have been related to the model parameters, and the full inverse procedure can now be established. These seven expressions are:

(1) Eq.(6), where unknown parameters include ρ_1 and c_{1p};

(2) Eq.(7), where unknown parameters include c_{2p} and d;

(3) Eq.(8), with one unknown parameter c_{1p};

(4) Eq.(9), with one unknown parameter c_{2p};

(5) Eq.(10), where unknown parameters include ρ_2 and c_{2p};

(6) Eq.(11), where unknown parameters include c_{2s} and d;

(7) Eq.(12), where unknown parameters include ρ_1, c_{1p} and c_{1s}.

With the above seven expressions, the seven unknown parameters of a seabed model will be recovered after some numerical analyses.

III. NUMERICAL RESULTS

A. Forward simulations on pulse reflection by the seabed model

In the forward problem, the tone-burst shown in Fig.2a is used as the incident signal. Two reflections are calculated, one is at normal incidence, and the other is at oblique incidence. Parameters used for the seabed model are: c_3=1500 m/s, ρ_3=1000 kg/m^3; c_{2p}=1677 m/s, c_{2s}=477 m/s, ρ_2=1830 kg/m^3, d=0.5 m; c_{1p}=3500 m/s, c_{1s}=1600 m/s, ρ_1=3000 kg/m^3.

At normal incidence, the total reflection coefficient with respect to the frequency is plotted in Fig.4. At oblique incidence, $\theta_3 = 25°$, the reflection coefficient is given in Fig.8.

B. Reconstruction of the seabed model

At normal incidence, from Fig.4, it is measured R_{max} to be 0.75, and Δf to be 1674 Hz; From Fig.5, the following data are derived: R_1=0.3433, $\alpha_1 = 25.4°$, $\alpha_2 = 63.4°$. At oblique incidence, Fig.8 gives: R_x=0.88 and δf=480 Hz.

By insertion of the above measured values into Eqs.(6-12), the seven parameters are recovered. These recovered values of these parameters are listed in the following table together with the assumed values.

Parameter	ρ_2 (kg/m^3)	c_{2p} (m/s)	c_{2s} (m/s)	d (m)	ρ_1 (kg/m^3)	c_{1p} (m/s)	c_{1s} (m/s)
Initial	1830	1677	477	0.5	3000	3500	1600
Recovered	1829	1678	476	0.5012	3003	3497	1581

The agreement between the recovered values and the assumed initial values is good, especially for the parameters recovered by the use of spectral measurements.

CONCLUSIONS

Numerical simulations and theoretical analyses have been made on the reflection of time domain signals by a multi-layer seabed model. An inverse procedure has been established for the reconstruction of a three-layer seabed model, and seven unknown parameters have been recovered. A good agreement between the recovered values and the assumed initial values of the seven parameters was obtained, especially for those parameters recovered by use of spectral measurements. Except for attenuation, all parameters necessary for description of the model seabed can be obtained by use of reflections at different angles of incidence.

The recovery of the shear wave velocities in the substrate layer is very sensitive to the result of the longitudinal wave velocity recovery in the same layer.

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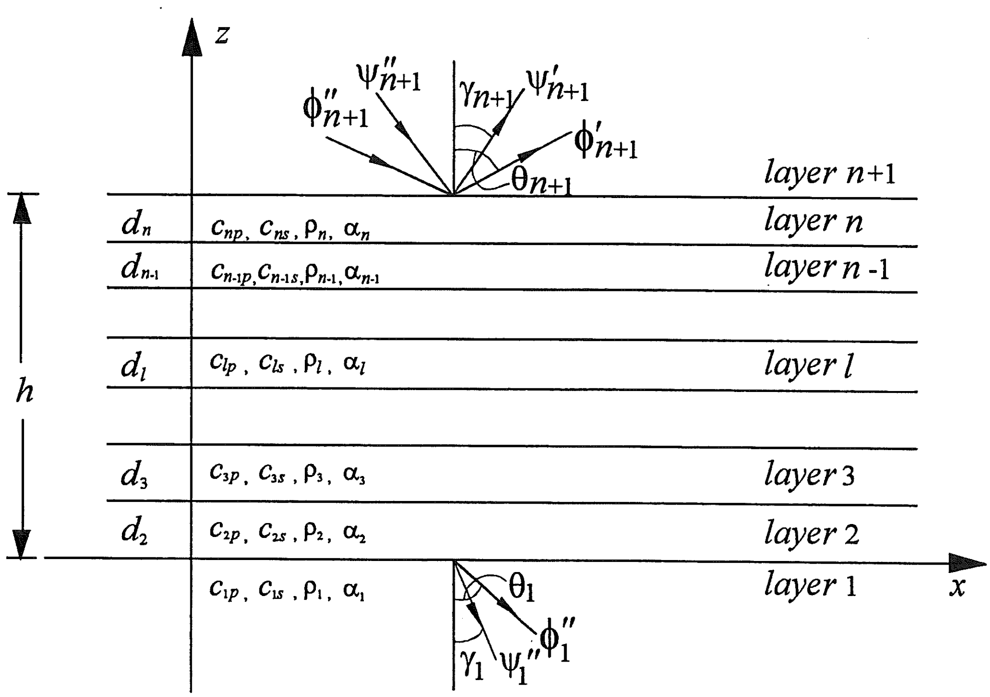

Fig.1. *Geometry of wave propagation in a multi-layer model and its parameters.*

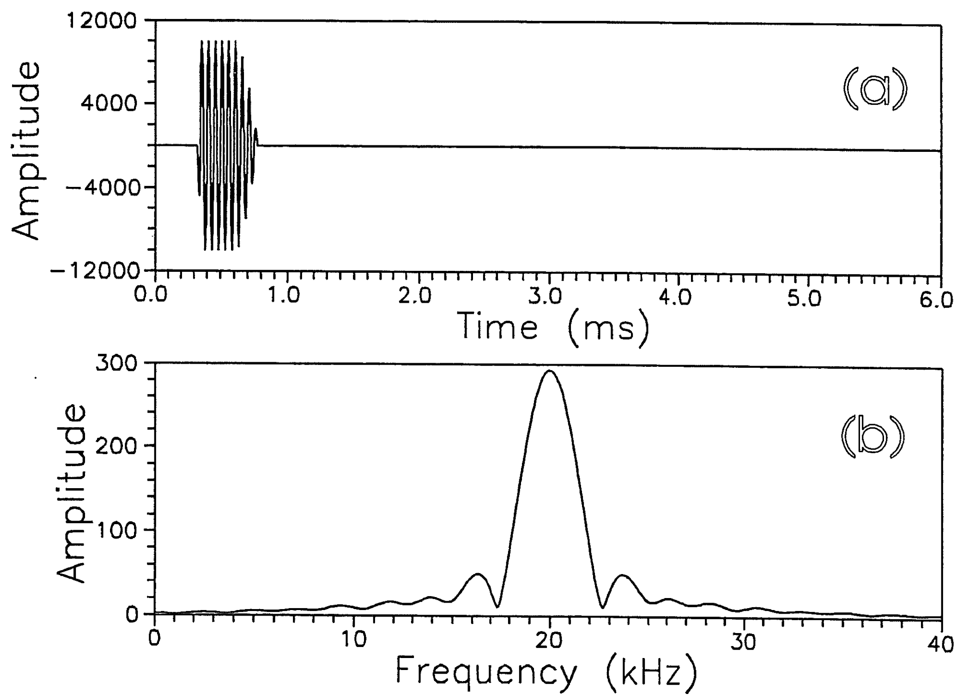

Fig.2. *Tone-burst used as the incident signal and its frequency spectrum.*

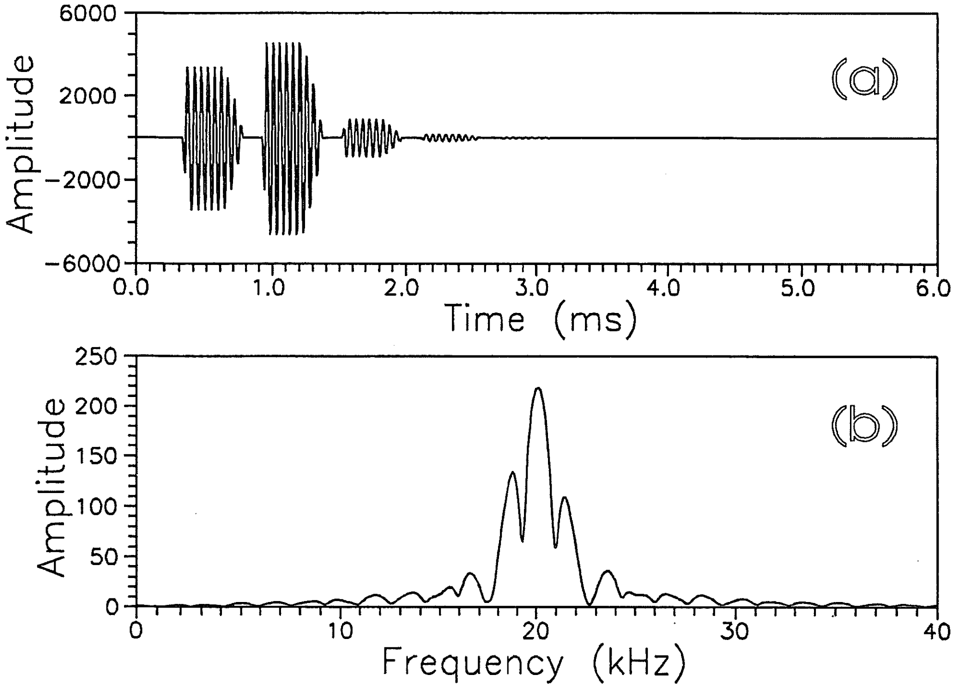

Fig.3. *The time signal reflected by a three-layer seabed model at normal incidence and its frequency spectrum.*

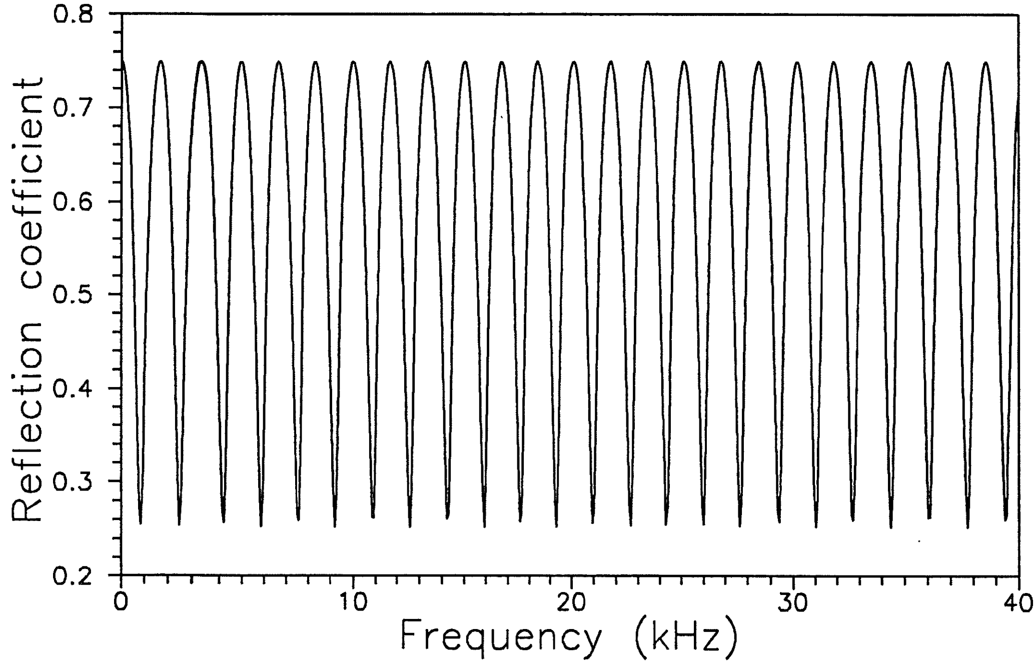

Fig.4. *Dependence of the reflection coefficient on frequency for a three-layer seabed model. Normal incidence.*

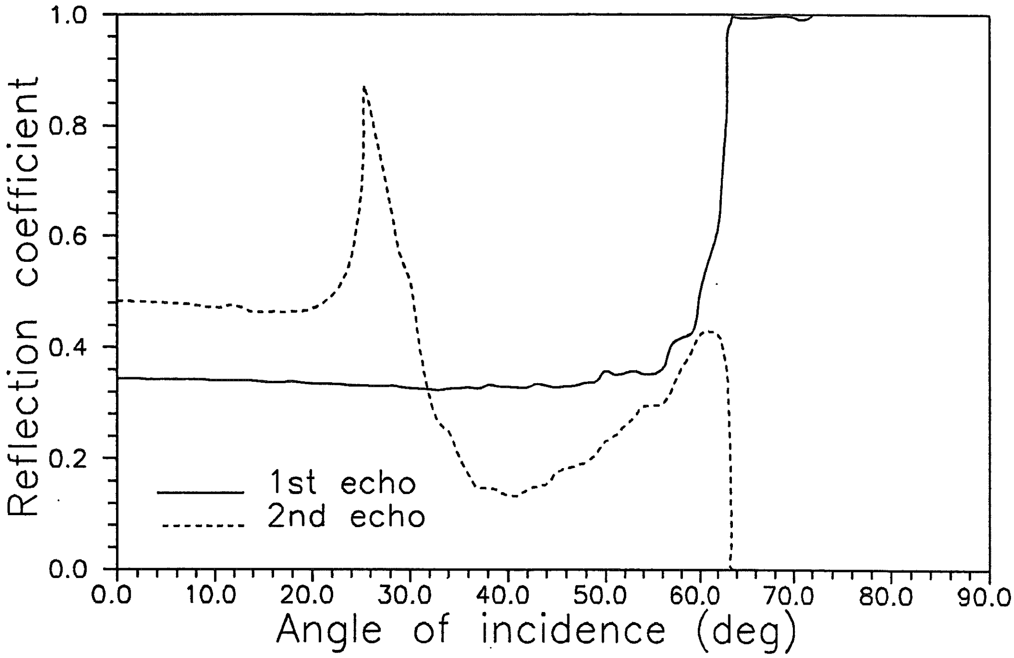

Fig.5. *Angle dependence of the first two reflected echoes from a three-layer seabed model including shear waves.*

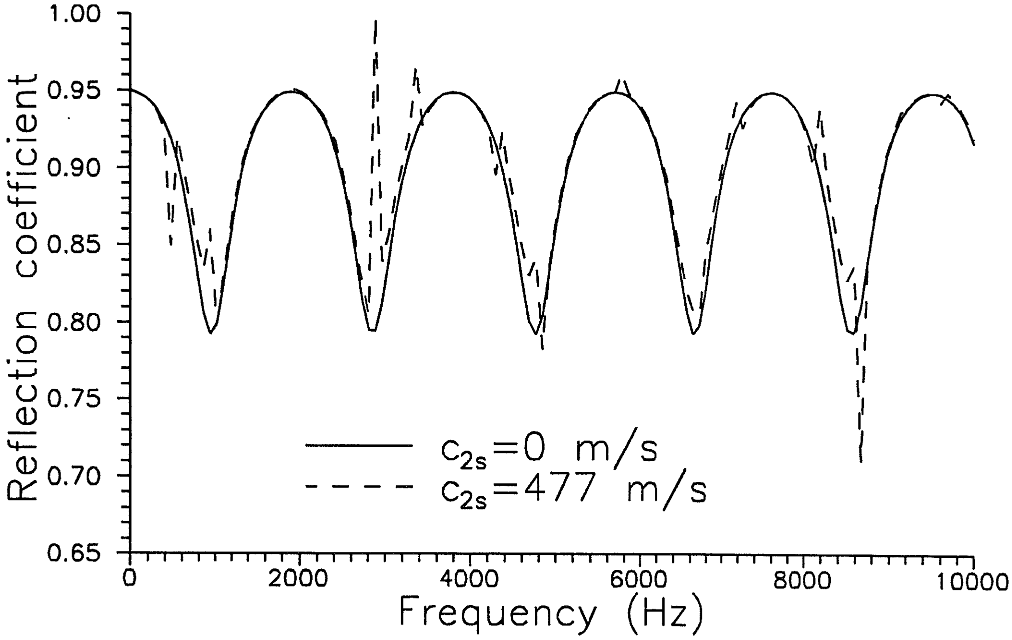

Fig.6. *Influence of shear waves in the sediment layer on the total reflection coefficient. Angle of incidence* $\theta_3 = 25°$.

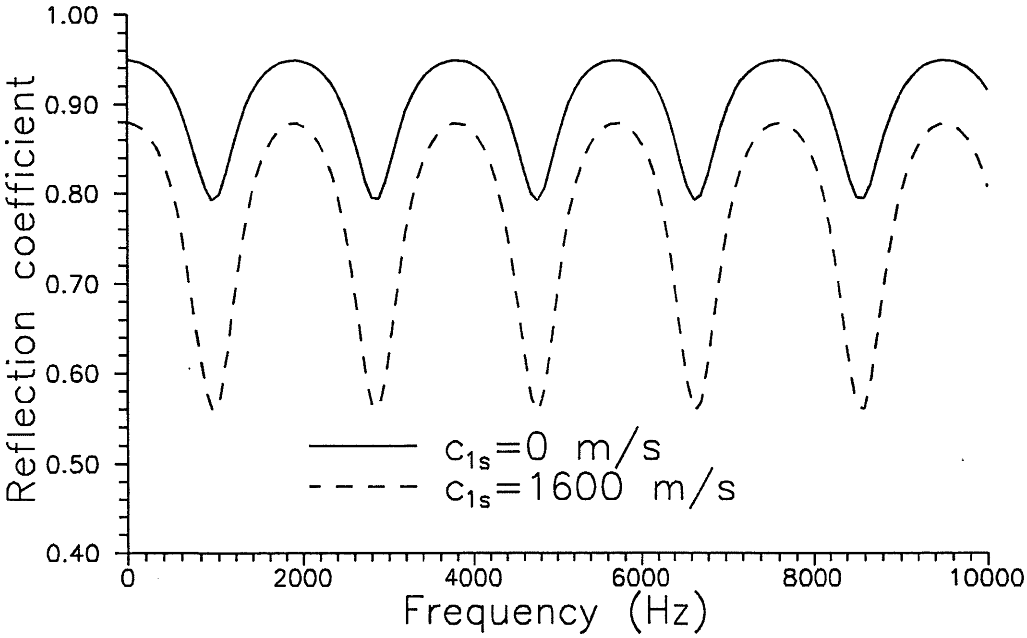

Fig.7. *Influence of shear waves in the substrate layer on the total reflection coefficient. Angle of incidence* $\theta_3 = 25°$.

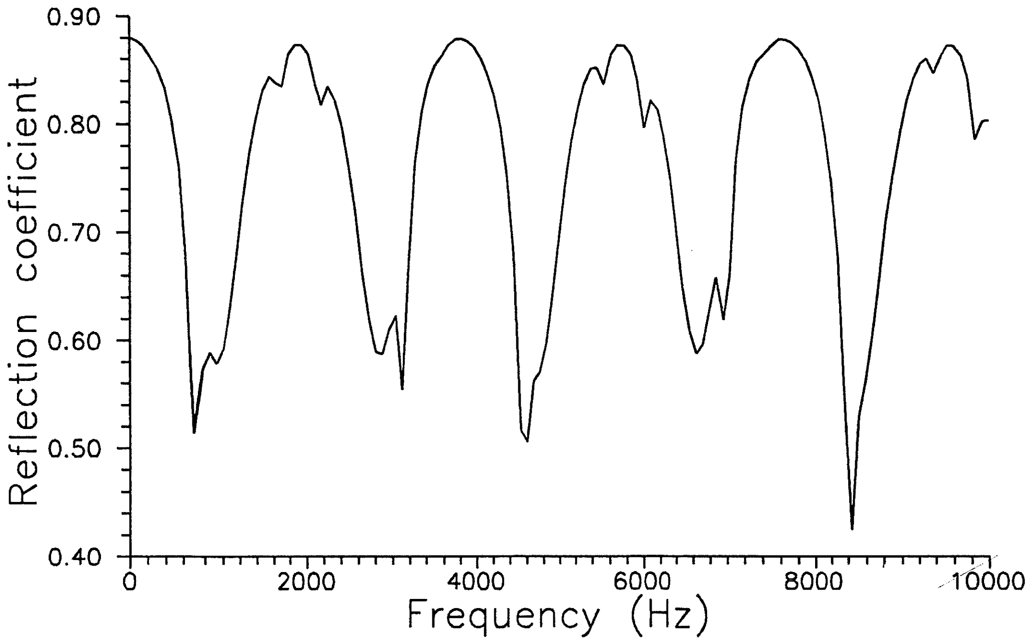

Fig.8. *Dependence of the total reflection coefficient on the frequency for a seabed model. Angle of incidence* $\theta_3 = 25°$.