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Analytic Diffraction Correction for the Accurate Experimental Modeling of Absorption and Dispersion Properties of Visco-Elastic Materials

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Abstract — This paper presents a system identification approach for the accurate parametric modeling of both reflection and transmission experiments performed on a linear homogeneous visco-elastic material at normal incidence. The plane wave propagation model takes into account the absorption and dispersion in the material as well as an analytic diffraction correction for the beam spread. The proposed inverse procedure is based on a maximum likelihood estimator, which is known to outperform most other estimators especially when the input/output measurements are heavily corrupted by noise. It is shown by means of a model validation that this approach leads to accurate estimates of the absorption and dispersion.

I. INTRODUCTION

Although different methods based on plane wave propagation models are available for the determination of the absorption and dispersion in a visco-elastic material at normal incidence [1], [2], [3], these methods do not allow a study of the influence of possible model errors. However, the estimation of the physical material properties from noisy reflection or transmission data is strongly influenced by the validity of the used wave propagation models as well as the applied estimation techniques. In this paper, a model validation is carried out for a plane wave propagation model including an analytic diffraction correction method. The inverse problem is solved by means of a maximum likelihood approach, which is formulated in the framework of frequency domain system identification [4]. In [4] the identification of parametric models for transmission experiments was discussed. A similar scheme is followed in this paper, consisting of the parametric modeling of the experiments and the estimation of the parameters involved in the analytic wave propagation model. The validity of the proposed model is demonstrated by means of a comparison between the estimated absorption and dispersion obtained with reflection and transmission data.

The parametric modeling of the reflection and transmission experiments performed on a linear visco-elastic material (e.g. plexiglass) at normal incidence is discussed in Section II. The modeling problem consists of two parts: the modeling of the ultrasonic wave propagation in the visco-elastic plate and the calibration of the measurement setup. An analytic plane wave propagation model is used to describe the reflection and transmission coefficients of a visco-elastic plate including the absorption and dispersion effects. Although in a classical approach the presence of absorption and dispersion in a visco-elastic material is described by models derived from a nearly local form of the Kramers-Kronig relationship [5], [6], [7], a rational transfer function model is used in this paper. This model choice is based on the conclusions drawn in [4], where it was shown that rational transfer function models result in broadband absorption-dispersion functions which describe the ultrasonic wave propagation much better than the classical models. Moreover, an analytic diffraction correction is introduced in the plane wave model based on the method proposed in [8]. Next, the calibration of the measurement setup is formulated for both the reflection and transmission experiments. This calibration requires two additional experiments performed on a reference system, from which a nonparametric estimate of the transfer characteristics of the measurement setup in reflection and transmission can be determined.

The calibration experiments together with the reflection or transmission experiments performed on the visco-elastic plate are regarded as the inputs, respectively, the outputs of a SISO (Single Input Single Output) system. This allows the construction of a SISO maximum likelihood estimator (Section III).

An experimental comparison between the estimated model parameters as well as their uncertainty obtained with the reflection and the transmission identification approach is carried out for a plexiglass plate in Section IV.

Special attention is paid to the analysis of the performed experiments and the obtained measurement accuracy after calibration. Moreover, a comparison is made between the estimated absorption and dispersion characteristics obtained with the plane wave propagation model without and with diffraction correction. Finally, the conclusions are drawn in Section V.

II. MODELING OF THE REFLECTION AND TRANSMISSION EXPERIMENTS

In this section, the modeling of reflection and transmission experiments performed at normal incidence on a visco-elastic plate immersed in water is treated in the framework of a SISO representation for the system under investigation. The measurement setup, the plane wave propagation model as well as the diffraction correction, and the calibration of the experiments are discussed separately in the following.

A. The measurement setup

In Fig. 1 the measurement setup for the reflection and transmission experiments is depicted. The experiments are performed with a pulser-receiver (Panametrics 5055PR) and broadband transducers (type Panametrics V389 with a center frequency of 500 kHz, used frequency band [300 kHz, 700 kHz], element diameter 3.81 cm and near field distance in water 12.1 cm). The acquisition of the reflection and transmission experiments is carried out using a classical pulse as excitation signal in the monostatic (reflection) and bistatic (transmission) mode, respectively. Consequently, the experiments are not performed simultaneously. The position of the transducers is not altered when switching from the monostatic to the bistatic measurement mode.

Since the inverse problem is based on a frequency domain maximum likelihood estimator, the measurement setup is described in the frequency domain too. The following notations are introduced: $H_{Ae}(\omega)$ for the emitter amplifier, $H_{Tie}(\omega)$ and $H_{Tir}(\omega)$ for the emitter and receiver, respectively with $i = 1, 2$, while $H_{Ar}(\omega)$ and $H_{At}(\omega)$ represent the receiving amplifier characteristics. The generated pulses $X_{mr}(\omega)$ and $X_{mt}(\omega)$ (inputs) as well as the reflected signal $Y_{mr}(\omega)$ and the transmitted signal $Y_{mt}(\omega)$ (outputs) are digitized. The input and output measurement errors are represented by the random variables $n_{Xi}(\omega)$ and $n_{Yi}(\omega)$, which have a frequency dependent variance $\sigma^2_{Xi}(\omega)$ and $\sigma^2_{Yi}(\omega)$, respectively with $(i = r, t)$.

Introducing the transfer functions of the measurement setup in reflection and transmission $H_{sysr}(\omega)$ and $H_{syst}(\omega)$, respectively, the following relationships are obtained in the frequency domain between the inputs and outputs of the system under investigation:

$$Y_{mr}(\omega) = H_{sysr}(\omega) \cdot H_r(\omega) \cdot D_r(\omega) \cdot X_{mr}(\omega) \tag{1}$$

$$Y_{mt}(\omega) = H_{syst}(\omega) \cdot H_t(\omega) \cdot D_t(\omega) \cdot X_{mt}(\omega) \tag{2}$$

where $H_r(\omega)$ and $H_t(\omega)$ represent, respectively, the plane wave propagation models for reflection and transmission in the visco-elastic plate. Moreover, the diffraction effects are taken into account by the introduction of the diffraction correction factors $D_r(\omega)$ and $D_t(\omega)$ for the reflection and transmission case, respectively. The analytic expressions for these correction factors are presented in Section C. From (1) and (2) it can be readily seen that calibration of the measurement setup, both for the reflection and transmission experiments, requires the knowledge of $H_{sysr}(\omega)$ and $H_{syst}(\omega)$, respectively, given by:

$$H_{sysr}(\omega) = H_{T1e}(\omega) \cdot H_{T1r}(\omega) \cdot H_{Ar}(\omega) \tag{3}$$

$$H_{syst}(\omega) = H_{T1e}(\omega) \cdot H_{T2r}(\omega) \cdot H_{At}(\omega) \tag{4}$$

Note that using the emitter as the receiver and vice versa (i.e. replacing subscript 1 by 2 and 2 by 1) affects the transfer functions (3) and (4), even in the case that both transducers remain at the same position. The calibration techniques for the reflection and transmission experiments are briefly summarized in Section D. In the following section, the transfer function models for the plane wave propagation are discussed.

B. The plane wave propagation model

The parametric modeling of the plane longitudinal wave propagation in a single layered structure immersed in water is discussed for reflection and transmission (Fig. 1). Assuming that the superposition principle is valid (which will be experimentally verified in section IV), the broadband acoustic excitation is decomposed into plane harmonic waves, while the material under investigation is modelled in the framework of linear visco-elastic theory.

Describing the absorption and dispersion in a linear visco-elastic homogeneous material by means of a rational transfer function model, the complex wave number $K_M(\omega)$ for the material under investigation is defined as follows:

$$K_M(\omega) = \frac{\omega}{V_M(\omega)} = \frac{\omega}{c(\omega)} - j \cdot \alpha(\omega) \tag{5}$$

with:

$$V_M(\omega) = c_{0M}\sqrt{\frac{1+\sum\limits_{l=1}^{n}\alpha_{Ml}(j\omega)^l}{1+\sum\limits_{l=1}^{d}\beta_{Ml}(j\omega)^l}} = c_{0M}\cdot P_M(\omega,\alpha_M,\beta_M) \tag{6}$$

$$\alpha_M = (\alpha_{M1},\ldots,\alpha_{Mn}) \tag{7}$$

$$\beta_M = (\beta_{M1},\ldots,\beta_{Md}) \tag{8}$$

The complex velocity $V_M(\omega)$ depends on the dispersionless phase velocity c_{0M} and the numerator and denominator coefficients α_M and β_M, respectively, of the rational form. Without loss in generality, the wave propagation in water is assumed to be lossless (no absorption and dispersion):

$$K_W(\omega) = \frac{\omega}{c_{0W}} \tag{9}$$

Expressing the boundary conditions for the normal stress and displacement at the 2 interfaces of the layered system shown in Fig. 1, the wave propagation models in reflection and transmission $H_r(\omega)$ and $H_t(\omega)$, respectively, are given by [9]:

$$H_r(\omega) = \frac{N_r(\omega)}{D_r(\omega)} = \frac{r_{WM}(\omega)e^{-2jK_W(\omega)d_1}\left(1-e^{-2jK_M(\omega)d_2}\right)}{D(\omega)} \tag{10}$$

$$H_t(\omega) = \frac{N_t(\omega)}{D_t(\omega)} = \frac{(1-r_{WM}^2(\omega))\,e^{-jK_W(\omega)\,(d_1+d_3)}\,e^{-jK_M(\omega)d_2}}{D(\omega)} \tag{11}$$

These transfer function models have a common denominator given by:

$$D(\omega) = D_r(\omega) = D_t(\omega) = 1-r_{WM}^2(\omega)e^{-2jK_M(\omega)d_2} \tag{12}$$

Furthermore, the reflection coefficient at the boundary water-material can be expressed as:

$$r_{WM}(\omega) = \frac{1-Z_{WM}\cdot P_M(\omega,\alpha_M,\beta_M)}{1+Z_{WM}\cdot P_M(\omega,\alpha_M,\beta_M)} \tag{13}$$

in which Z_{WM} is defined as the ratio of the acoustic impedances of the material under investigation and water in the case no absorption and dispersion would be present:

$$Z_{WM} = \frac{\rho_M c_{0M}}{\rho_W c_{0W}} \tag{14}$$

and where ρ_W and ρ_M denote respectively the density of water and of the material under investigation.

Note that for physical reasons (stability and $c(\omega), \alpha(\omega) \geq 0, \forall\omega$), constraints are introduced on the numerator and denominator coefficients α_M and β_M, respectively, of the rational form [4].

C. The analytic diffraction correction

In order to introduce the analytic diffraction correction in the reflection and transmission coefficients of the plate, the closed form expression for the plane wave reflection and transmission coefficients (10) and (11) are replaced by a summation known as the Debye series expansion corresponding with the multiple propagation paths in the layer [9]. Performing a summation over all propagation paths, the respective reflection and transmission coefficients $H_{Dr}(\omega)$ and $H_{Dt}(\omega)$ corrected for diffraction effects are given by:

$$\begin{aligned} H_{Dr}(\omega) &= D_r(\omega) \cdot H_r(\omega) \\ &= r_{WM}(\omega) e^{-2jK_W(\omega)d_1}\left(D_{ro}(\omega) - (1 - r_{WM}^2(\omega)) \sum_{n=1}^{\infty} R_n(\omega)\right) \end{aligned} \tag{15}$$

$$\begin{aligned} H_{Dt}(\omega) &= D_t(\omega) \cdot H_t(\omega) \\ &= (1 - r_{WM}^2(\omega))\, e^{-jK_W(\omega)(d_1 + d_3)} e^{-jK_M(\omega)d_2}\left(\sum_{n=0}^{\infty} T_n(\omega)\right) \end{aligned} \tag{16}$$

where:

$$R_n(\omega) = D_{rn}(\omega) \cdot [r_{WM}(\omega)]^{(2n-2)} \cdot e^{-2jnK_M(\omega)d_2} \tag{17}$$

and:

$$T_n(\omega) = D_{tn}(\omega) \cdot [r_{WM}(\omega)]^{2n} \cdot e^{-2jnK_M(\omega)d_2} \tag{18}$$

Note that the correction factors $D_{rn}(\omega)$ and $D_{tn}(\omega)$ are introduced for each propagation path through the plate for reflection and transmission, respectively. By setting the correction factors equal to 1, the classical plane wave reflection and transmission coefficients are generated again. In practice, the infinite summations appearing in (15) and (16) are reduced to finite ones due to the presence of absorption in the material under investigation. The expressions for the diffraction correction factors for reflection are given by [8]:

$$D_{rn}(\omega) = 1 - e^{-jS_{rn}}[J_0(S_{rn}) + jJ_1(S_{rn})] \tag{19}$$

where the arguments:

$$S_{rn} = \frac{\omega a^2}{2d_1 c_{0W} + 2nd_2 c_{0M}} \tag{20}$$

depend on the frequency independent phase velocities c_{0W} and c_{0M}, the thickness of the plate d_2, the distance d_1 (see Fig. 1) as well as on the radius a of the used transducer. Similarly, the diffraction correction factors for the transmission experiment are obtained as follows [8]:

$$D_{tn}(\omega) = 1 - e^{-jS_{tn}}[J_0(S_{tn}) + jJ_1(S_{tn})] \tag{21}$$

where the arguments:

$$S_{tn} = \frac{\omega a^2}{(d_{tot} - d_2)\, c_{0W} + (2n+1)\, d_2 c_{0M}} \tag{22}$$

depend also on the distance $d_{tot} = d_1 + d_2 + d_3$ (see Fig. 1).

D. Calibration of the experiments

In general, the characteristic of the measurement setup is removed from the experiments performed on the material under investigation using additional measurement data. Thereto the material under investigation is replaced by a reference system, for which the wave propagation can be described by means of an analytic model, while the experimental conditions (e.g. pulser-receiver configuration, positioning, temperature) are maintained. In this section, the used calibration methods for the transmission and reflection experiments are discussed by treating the modeling of the calibration experiments in a similar way as the experiments performed on the test sample. Use is made of the general assumption that water behaves lossless in the frequency band of interest (300 upto 700 kHz).

Calibration experiment for transmission. The calibration of the transmission experiments is elaborated as described in [4]. Removing the material under investigation from the measurement setup depicted in Fig. 1, the calibration experiment is performed without changing the pulser-receiver settings nor the positioning of the transducers. Using relationship (2), the transfer function for the calibration experiment in transmission $H_{calt}(\omega)$ can be introduced as follows:

$$\begin{aligned} Y_{calt}(\omega) &= D_{calt}(\omega) \cdot H_{syst}(\omega) \cdot H_{to}(\omega) \cdot X_{calt}(\omega) \\ &= H_{calt}(\omega) \cdot X_{calt}(\omega) \end{aligned} \tag{23}$$

with:

$$H_{to}(\omega) = e^{-jK_W(\omega)d_{tot}} \tag{24}$$

Since it is assumed that water behaves lossless, the analytic model $H_{to}(\omega)$ describes simply the time delay corresponding with the propagation distance d_{tot} between the emitting and the receiving transducers. The diffraction correction factor for the calibration experiment $D_{calt}(\omega)$ equals [8]:

$$D_{calt}(\omega) = 1 - e^{-jS_{calt}}[J_0(S_{calt}) + jJ_1(S_{calt})] \tag{25}$$

where the argument S_{calt} now reduces to:

$$S_{calt} = \frac{\omega a^2}{d_{tot}c_{0W}} \tag{26}$$

Calibration experiment for reflection. Returning to equation (1), the calibration of the reflection experiments is modeled by means of the transfer function $H_{calr}(\omega)$:

$$\begin{aligned} Y_{calr}(\omega) &= D_{calr}(\omega) \cdot H_{sysr}(\omega) \cdot H_{ro}(\omega) \cdot X_{calr}(\omega) \\ &= H_{calr}(\omega) \cdot X_{calr}(\omega) \end{aligned} \tag{27}$$

where $H_{ro}(\omega)$ represents the plane wave propagation model for the chosen reference system and $D_{calr}(\omega)$ the diffraction correction factor. In the case where a calibration experiment is performed on a material with well known properties, the modeling of this calibration experiment is described using (1) and (15) in analogy with the reflection experiment performed on the material under investigation. In general, multiple reflections are created in the calibration sample so that relationship (15) can not be simplified. However, in the case the multiple reflections can be separated in the time domain, only the first reflection can be used by applying appropriate windowing in the time domain. Introducing the frequency dependent reflection coefficient $r_{WM0}(\omega, \boldsymbol{P}_o)$ of the interface water-calibration sample, the plane wave propagation model in calibration is given by:

$$H_{ro}(\omega) = e^{-2jK_W(\omega)d_{cal}} \cdot r_{WM0}(\omega, \boldsymbol{P}_o) \tag{28}$$

where $r_{WM0}(\omega, \boldsymbol{P}_o)$ is given by (13) and the vector $\boldsymbol{P}_o = \{Z_{WM0}, \alpha_{M0}, \beta_{M0}\}$ contains the model parameters of the calibration sample. Again, the diffraction correction factor for the calibration experiment in reflection is introduced as follows [8]:

$$D_{calr}(\omega) = 1 - e^{-jS_{calr}}[J_0(S_{calr}) + jJ_1(S_{calr})] \quad (29)$$

where the argument:

$$S_{calr} = \frac{\omega a^2}{2d_{cal}c_{0W}} \quad (30)$$

depends now on the distance d_{cal} between the transducer and the calibration sample.

Since this calibration approach requires the knowledge of $\boldsymbol{P_o}$ and since it is anyway assumed that the first reflection can be separated in time domain from the multiple reflections created in the calibration sample, an obvious and optimal choice of a calibration sample is the material under investigation itself. This choice is optimal with respect to the possible introduction of systematic errors through the estimation of the model parameters $\boldsymbol{P_o}$ by means of a transmission experiment. Indeed, a transmission experiment performed on the calibration sample is the only possibility to obtain these parameters without a priori knowledge as can be concluded from (23) and (24). Note that if the first reflection on the material under investigation is used for calibration purposes, the following substitutions can be introduced in (28):

$$d_{cal} = d_1 \qquad r_{WM0} = r_{WM} \quad (31)$$

Moreover, the diffraction correction factor $D_{ro}(\omega)$ as defined in (15) reduces to $D_{ro}(\omega) = D_{calr}(\omega)$.

Formulation of the calibration approach. In order to incorporate the calibration procedures in the identification scheme, the transfer function models derived for the calibration experiments (see (23) and (27)) are combined with the models for the reflection and transmission experiments on the material under investigation (see (1) and (2)). Elimination of the characteristics of the measurement setup leads to:

$$\frac{Y_{mr}(\omega)}{X_{mr}(\omega)} = H_{mr}(\omega) = \tilde{H}_{Dr}(\omega) \cdot H_{calr}(\omega) = \tilde{H}_{Dr}(\omega) \cdot \frac{Y_{calr}(\omega)}{X_{calr}(\omega)} \quad (32)$$

$$\frac{Y_{mt}(\omega)}{X_{mt}(\omega)} = H_{mt}(\omega) = \tilde{H}_{Dt}(\omega) \cdot H_{calt}(\omega) = \tilde{H}_{Dt}(\omega) \cdot \frac{Y_{calt}(\omega)}{X_{calt}(\omega)} \quad (33)$$

where $\tilde{H}_{Dr}(\omega)$ and $\tilde{H}_{Dt}(\omega)$ are defined as follows:

$$\tilde{H}_{Dr}(\omega) = \frac{H_{Dr}(\omega)}{H_{ro}(\omega)} = 1 - (1 - r_{WM}^2(\omega)) \sum_{n=1}^{\infty} R_n(\omega) \tag{34}$$

$$\tilde{H}_{Dt}(\omega) = \frac{H_{Dt}(\omega)}{H_{to}(\omega)} = (1 - r_{WM}^2(\omega))\, e^{jK_W(\omega)d_2} e^{-jK_M(\omega)d_2} \left(\sum_{n=0}^{\infty} T_n(\omega) \right) \tag{35}$$

In (34) and (35), the diffraction correction factors $D_{rn}(\omega)$ and $D_{tn}(\omega)$, as defined in (19) and (21), are replaced by:

$$\tilde{D}_{rn}(\omega) = \frac{D_{rn}(\omega)}{D_{calr}(\omega)} \text{ and } \tilde{D}_{tn}(\omega) = \frac{D_{tn}(\omega)}{D_{calt}(\omega)} \tag{36}$$

Finally, the arguments of the exponential functions appearing in (34) and (35) are written in the form:

$$K_W(\omega)d_2 = \omega\frac{d_2}{c_{0W}} = \omega\tau_T \text{ and } K_M(\omega)d_2 = \frac{\omega\tau_M}{P_M(\omega, \alpha_M, \beta_M)} \tag{37}$$

where the delay τ_T only occurs for the transmission model and originates from the difference in propagation time during the calibration and the transmission experiment, while τ_M represents the propagation time of the material under investigation. The common model parameters for the reflection and transmission modeling, which are purely related to the material under investigation, are summarized as follows: the propagation time τ_M, the ratio of the acoustic impedances Z_{WM} and the numerator and denominator coefficients α_M and β_M, respectively, of the rational form. Assuming that the density ρ_W and the phase velocity c_{0W} of water are known, the physical parameters of the material under investigation are easily derived from the introduced independent model parameters $\boldsymbol{P} = \{\tau_T, \tau_M, Z_{WM}, \alpha_M, \beta_M\}$ [4]. In the next section the estimation of the model parameters $\boldsymbol{P}$ from the noisy experiments is elaborated in the framework of a ML approach.

Since the calibrated reflection and transmission models defined in (33) and (34) relate transfer functions, special care should be taken to the nonparametric estimation of these transfer functions from the noisy input-output measurements. Clearly, stochastic as well as systematic errors should be minimized in order to obtain 'good' estimates for the model parameters.

III. THE ESTIMATION OF THE MODEL PARAMETERS

The estimation of the model parameters $\boldsymbol{P}$ is elaborated using a Maximum Likelihood (ML) approach for SISO systems (reflection or transmission data). In [11] the identification of transfer function models for linear SISO systems is treated in ML sense. A similar approach was followed for the identification of parametric models for transmission experiments [4]. In this section, a brief overview of the SISO identification approach is given.

The MLE for linear SISO systems constructed in [10], [11] requires the measured input and output spectra of the system under investigation as well as the knowledge of the perturbing noise variances. Furthermore, the noise sources are assumed to be zero mean complex normally distributed. In the case the measured spectra of input and output are obtained using a Discrete Fourier Transform this assumption is asymptotically valid [12]. Although, the MLE presented here for the reflection or transmission models (33) and (34) combines the calibration of the measurement setup as well as the transmission or reflection experiments on the material under investigation, the basic assumptions necessary for the ML approach are still valid in this case [4].

Introducing the nonparametric estimates $H_j^{np}(\omega)$ (the suffix j equals respectively *calr, calt* for the calibration experiments and *mr, mt* for the reflection or transmission experiments with test sample), the equations (33) and (34) can be transformed in a SISO representation of a linear system:

$$H_{mr}^{np}(\omega) = \tilde{H}_{Dr}(\omega, \boldsymbol{P}_r) \cdot H_{calr}^{np}(\omega) \tag{38}$$

$$H_{mt}^{np}(\omega) = \tilde{H}_{Dt}(\omega, \boldsymbol{P}_t) \cdot H_{calt}^{np}(\omega) \tag{39}$$

The SISO parameter vectors $\boldsymbol{P}_r = \{\tau_M, Z_{WM}, \alpha_M, \beta_M\}$ and $\boldsymbol{P}_t = \{\tau_T, \tau_M, Z_{WM}, \alpha_M, \beta_M\}$ are introduced for the sake of convenience. The nonparametric estimates occurring in (39) and (39) are obtained from averaging the input-output measurements during the calibration experiments and the reflection and transmission experiments [4]. During the averaging procedure the variances on the nonparametric estimates $\sigma_{H_j}^2(\omega)$ are determined as well. Due to the high signal-to-noise ratio of the input and output spectra (typically 40 dB), the $H_{\log}$ estimates will be almost complex normally distributed, so that also the assumptions required by the MLE are well fulfilled. Taking into account these considerations, the ML cost functions C_r, C_t to be minimized for the SISO system are given by [4]:

$$C_i(\boldsymbol{P}_i) = \sum_{l=1}^{L} |e_i(\omega_l, \boldsymbol{P}_i)|^2 \text{ with } i = r, t \tag{40}$$

where

$$e(\omega_l, P) = \frac{X_m(\omega_l) \cdot H(\omega_l, P) - Y_m(\omega_l)}{\sqrt{\sigma^2_{X_m}(\omega_l) \cdot |H(\omega_l, P)|^2 + \sigma^2_{Y_m}(\omega_l)}} \text{ with } i = r, t \tag{41}$$

and:

$$V_{e_i}(\omega_l, P_i) = \sigma^2_{H^{np}_{cal\,i}}(\omega_l) \cdot |\tilde{H}_i(\omega_l, P_i)|^2 + \sigma^2_{H^{np}_{mi}}(\omega_l) \tag{42}$$

where $\{\omega_l, l = 1, \ldots, L\}$ denotes the set of angular frequencies taken into account. The nonparametric estimates of the inputs and the outputs as well as their variances are determined at each spectral line l as explained before. Finally, $\tilde{H}_i(\omega_l, P_i)$ $(i = r, t)$ represent the transfer function models as defined in (34) and (35), respectively, evaluated at the l-th spectral line. Relationship (40) results in a nonlinear minimization problem. The Levenberg-Marquardt minimization algorithm is used because of it's convergence properties [15].

The inverse procedure is initialized for the plane wave propagation model (i.e. with $\tilde{D}_{rn}(\omega) = 1$ and $\tilde{D}_{tn}(\omega) = 1$). The initial parameter values for the model parameters P are obtained from the physical parameters of the material (thickness of the specimen, density, and dispersionless phase velocity) determined from basic experiments. Initial values for the numerator and denominator coefficients of the rational transfer function are not available. However, this problem is solved by starting the estimation only with a first order model $(n = 1, d = 0)$ or $(n = 0, d = 1)$, and selecting a very small parameter value [4]. After minimizing (40), the model order is gradually increased and the previously obtained estimation results are used as initial parameter values, while the new added coefficient again is chosen very small. Since the initial parameter values of the other parameters are determined within a measurement accuracy of 10%, the minimization does not suffer from the presence of local minima. The next step in the estimation scheme consists of introducing the diffraction correction factors calculated from the estimates of P obtained with the plane wave propagation model. Since the correction factors (see (20), (22), (26) and (30)) depend on the radius of the transducer a (provided by the manufacturer), the phase velocity in water c_{0W} (assumed to be known given the temperature [14]), the distances d_1 and d_{tot}, and finally the thickness of the plate d_2 and the phase velocity c_{0M}, it is expected that the correction factors $\tilde{D}_{rn}(\omega)$ and $\tilde{D}_{tn}(\omega)$ can be calculated accurately from the previously obtained estimates of P with the plane wave model. Note that the followed procedure can be applied iteratively in order to increase the accuracy of the introduced correction factors.

IV. THE EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results presented in this section are obtained from reflection and transmission experiments performed on a plexiglass plate. The reflection experiments are calibrated using the first reflection on the plexiglass plate as explained in Section II.C. A comparison is made between the estimated absorption and dispersion properties of the plexiglass resulting from the SISO estimations obtained with the plane wave propagation model without and with diffraction correction.

The identification scheme presented in Section III is applied to the reflection and transmission experiments. Firstly, the model order selection is carried out using the plane wave propagation model without diffraction correction. In Table I, the material properties of plexiglass are summarized which are used as initial values for the estimations. From the cost function values presented in Table II, it is concluded that for the SISO reflection as well as the SISO transmission case the $(n = 2, d = 1)$ model order should be preferred. Indeed, a significant decrease of the cost function values is observed in comparison with the $(n = 1, d = 1)$ model order. A further increase of the model order to $(n = 2, d = 2)$ resulted in the same cost function as obtained for the $(n = 2, d = 1)$ model order. Next, the diffraction correction is introduced and the model order selection is repeated. Again, the $(n = 2, d = 1)$ model is selected. From Table II it also follows that the cost function values obtained for the model with diffraction correction are smaller than those obtained with the plane wave model. In Figs. 2 and 3 the quality of the SISO estimation results for transmission and reflection, respectively, are illustrated for the $(n = 2, d = 1)$ model with diffraction correction. Although no systematic errors are observable from the magnitude and phase differences (Fig. 2(c), (d) and Fig. 3(c), (d)) for model order $(n = 2, d = 1)$, a comparison between the value of the experimental cost functions ($C_r = 965$ and $C_t = 2002$) and the theoretical ones ($C_r \equiv C_t = 163$; with $L = 166$ and where $n_{pr} = 5$ and $n_{pt} = 6$ represent the number of parameters to be estimated in reflection and transmission, respectively [15]) leads to the conclusion that still small model errors are present. The estimated parameters as well as their uncertainty are given in Table IV and IV for the plane wave propagation model without and with diffraction correction, respectively. From these estimation results, it is concluded that the difference between the estimated model parameters obtained from the transmission and reflection case, respectively, is significantly smaller for the plane wave model including diffraction correction as compared to the model without diffraction correction. Although, the estimates cannot be considered equal within their uncertainty, it is concluded that only small model errors are present.

The physical relevance of the introduced diffraction correction is illustrated by comparing the estimated absorption and dispersion characteristics obtained with the plane wave propagation model without and with diffraction correction (Figs. 4 and 5). It is readily observed that without diffraction correction, the estimated absorption and dispersion are over-estimated, and that the reflection and transmission estimates lead to different characteristics. This is explained by the difference in calibration for both types of experiments, which results in a different contribution of the beam spread on the estimates. Introducing the diffraction correction, however, shows that a very good agreement exists between the reflection and transmission estimates of the absorption and dispersion. This result confirms the validity of the model including the diffraction correction.

Finally, the estimations are repeated by introducing the diffraction correction factors calculated from the previously obtained estimates. After convergence, this results in cost function values which are almost identical as those shown in Table II (< 1% of difference). It follows that the correction factors are not sensitive to the variations in the parameters P_r and P_t with respect to their initial values obtained with the plane wave model.

V. CONCLUSIONS

In this paper, a frequency domain identification approach is presented for reflection and transmission experiments performed on linear visco-elastic materials at normal incidence. Maximum likelihood estimators are developed for the estimation of the model parameters from noisy reflection and transmission data. The ultrasonic wave propagation is modelled by means of a plane wave theory without and with diffraction correction. For both models, the absorption and dispersion in the material under investigation is described using a rational transfer function model. Furthermore, special attention is paid to incorporate the calibration of the measurement setup in the identification scheme.

It is demonstrated that applying the presented approach to reflection and transmission experiments performed on a plexiglass plate, leads to the same model orders for the rational transfer function model for both wave propagation models. However, only the model including diffraction correction results in absorption and dispersion characteristics in very good agreement for the SISO reflection and SISO transmission estimations. These estimation results confirm the validity of the wave propagation model including the absorption and dispersion (rational transfer function model) as well as the analytic diffraction correction.

REFERENCES

[1] Mauro Bramanti, 'A Nondestructive Diagnostic Method Based on Swept-Frequency Ultrasound Transmission-Reflection Measurements', *IEEE Trans. Instr. & Meas.*, Vol. 41, No. 4, pp. 490-494, August 1992.

[2] Chin C. Lee, Mike Lahham and B.G. Martin, 'Experimental Verification of the Kramers-Kronig Relationship for Acoustic Waves', *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.*, Vol. 37, No. 4, pp. 286-294, July 1990.

[3] G. Guidarelli, A. Marini and L. Palmeiri, 'Ultrasonic Method for Determing Attenuation Coefficients in Plate-Shaped Materials', *J. Acoust. Soc. Am.*, Vol. 94, No. 3, pp.1476-1481, Sept. 1993.

[4] L. Peirlinckx, R. Pintelon and L. Van Biesen, 'Identification of Parametric Models for Ultrasonic Wave Propagation in the Presence of Absorption and Dispersion', *IEEE Trans. Ultrason. Ferroelec. Freq. Contr.*, Vol. 40, No. 4, pp. 302-312, July 1993.

[5] M. O'Donnell, E.T. Jayes and J.G. Miller, 'Kramers-Kronig Relationship between Ultrasonic Attenuation and Phase Velocity', *J. Acoust. Soc. Am.*, Vol. 69, No. 3, pp. 696-701, March 1981.

[6] Walter I. Futterman, 'Dispersive Body Waves', *J. Geophysical Research* , Vol. 67, No. 13 pp. 5279-5291, Dec. 1962.

[7] Einar Kjartansson, 'Constant Q-Wave Propagation and Attenuation', *J. Geophysical Research*, pp. 4737-4748, Aug., 1979.

[8] R.B. Thompson & T.A. Gray, 'Analytic Diffraction Corrections to Ultrasonic Scattering Measurements', in *Review of Progress in Quantitative Nondestructive Evaluation 2*, ed. D.O. Thompson & D.E. Chimenti, pp. 567-586, Plenum, New York, 1983.

[9] L.M. Brekhovskikh, *Waves in Layered Media*, Academic Press, New York, 1980.

[10] J. Schoukens & R. Pintelon, *Identification of Linear Systems. A Practical Guideline to Accurate Modeling.* Pergamon Press, 1991.

[11] R. Pintelon & J. Schoukens, 'Robust identification of Transfer Functions in the s- and z-Domains', *IEEE Trans. Instrum. Meas.*, vol. IM-39, No. 4, pp. 565-573, 1990.

[12] J. Schoukens & J. Renneboog, 'Modeling the Noise Influence on the Fourier Coefficients After a Discrete Fourier Transform', *IEEE Trans. Instrum. Meas.*, vol. IM-35, pp. 278-286, Sept. 1986.

[13] J.J. Moré, 'The Levenberg-Marquardt Algorithm: Implementation and Theory', *Numerical Analysis*, ed. G.A. Watson, Lecture Notes in Mathematics 630, Springer-Verlag, pp. 105-116, 1977.

[14] J.R. Lovett, 'Comments Concerning the Determination of Absolute Sound Speeds in Distilled and Seawater', *J. Acoust. Soc. Am.*, Vol. 45, pp.1051-1052, 1969.

[15] L. Peirlinckx, P. Guillaume, R. Pintelon and L. Van Biesen, 'MIMO Identification of Parametric Models for Ultrasonic Reflection and Transmission Experiments.', *Proceedings of the IEEE Ultrasonic Symposium*, Baltimore, Vol. 2, pp. 723–728, October 31 – November 3, 1993.

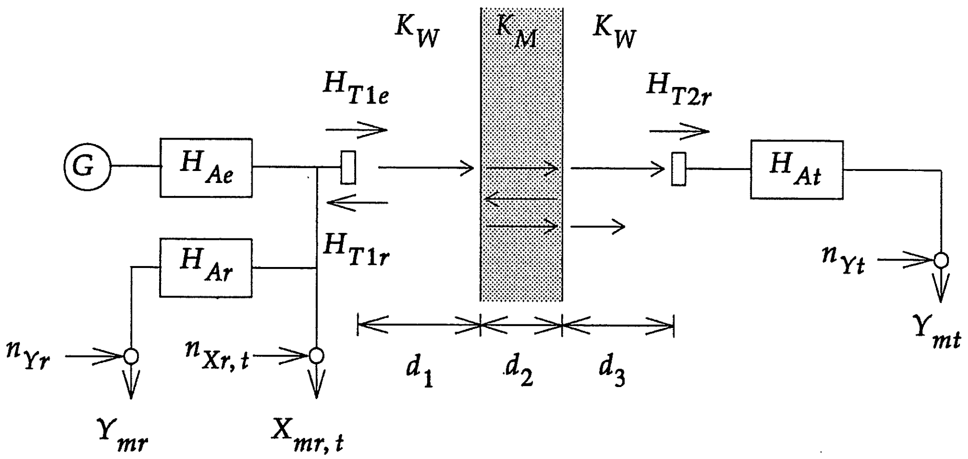

FIGURE 1: The measurement setup for the reflection experiment (monostatic mode) and the transmission experiment (bistatic mode).

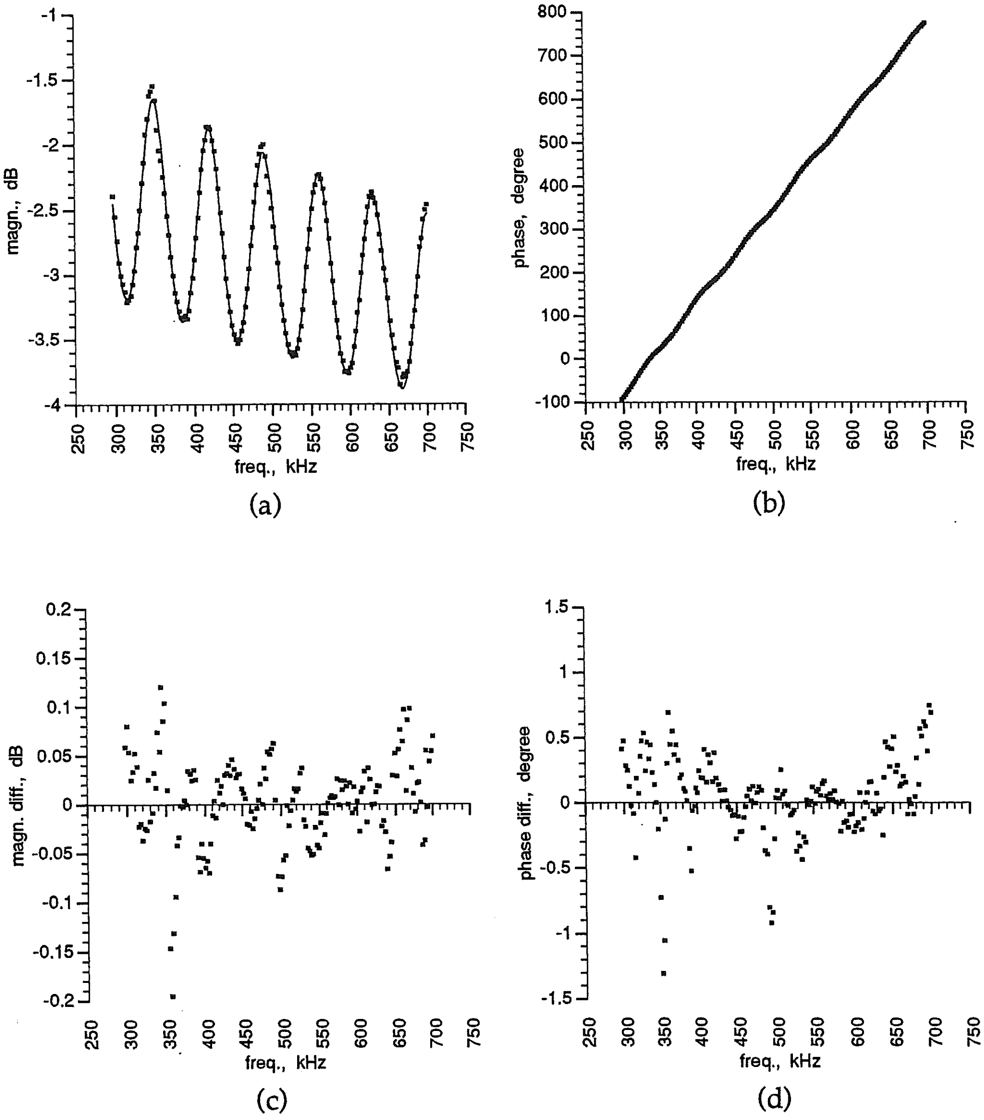

FIGURE 2: SISO estimation results in transmission for plexiglass. (a) The magnitude of the measured (■) and estimated (—) transfer function. (b) The phase of the measured (■) and estimated (—) transfer function. (c) The magnitude difference between the estimated and measured transfer function (■). (d) The phase difference between the estimated and measured transfer function (■).

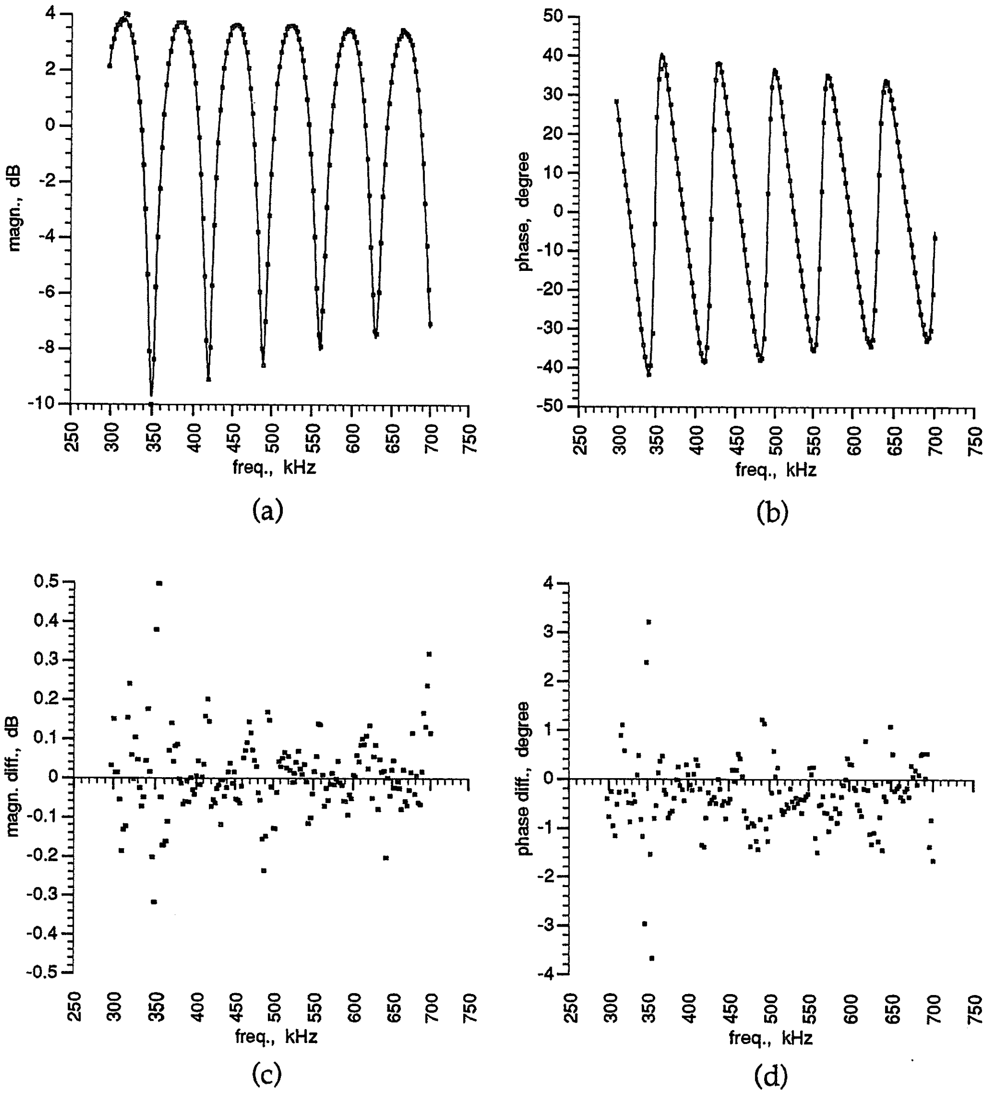

FIGURE 3: SISO estimation results in reflection for plexiglass.
(a) The magnitude of the measured (■) and estimated (–) transfer function. (b) The phase of the measured (■) and estimated (–) transfer function. (c) The magnitude difference between the estimated and measured transfer function (■). (d) The phase difference between the estimated and measured transfer function (■).

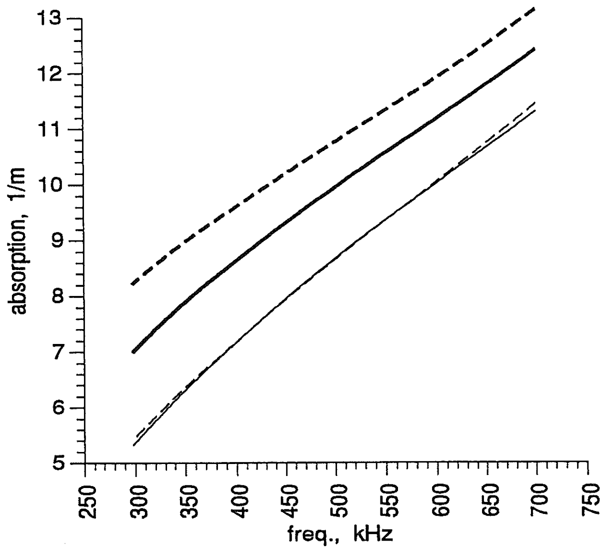

FIGURE 4: Comparison between the estimated absorption obtained with: plane wave propagation model in reflection (thick dashed line) and in transmission (thick line), plane wave model with diffraction correction in reflection (thin dashed line) and in transmission (thin line) (model order of the rational form $(n = 2, d = 1)$).

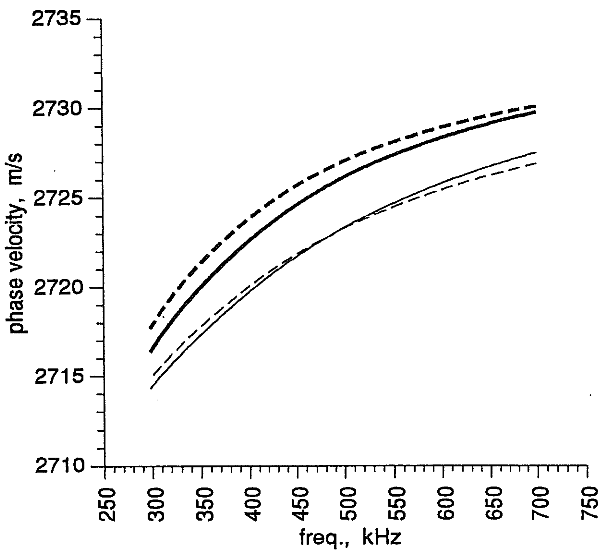

FIGURE 5: Comparison between the estimated dispersion obtained with: plane wave propagation model in reflection (thick dashed line) and in transmission (thick line), plane wave model with diffraction correction in reflection (thin dashed line) and in transmission (thin line) (model order of the rational form $(n = 2, d = 1)$).

TABLE I: Material properties: Sample thickness, dilatational and shear velocity at $500kHz$ and the density.

material	sample thickness (m)	dilatational velocity (ms^{-1})	shear velocity (ms^{-1})	density (kgm^{-3})
plexi	0.021	2700	1400	1200

TABLE II: Overview of the cost function values obtained for different model orders.

plane wave model	without diffraction correction		with diffraction correction	
model order	1 - 1	2 - 1	1 - 1	2 - 1
SISO reflection	1927	969	1534	965
SISO transmission	6310	2214	4006	2002

TABLE III: Estimated parameters and uncertainty (68% confidence) for plexiglass obtained with the SISO approach for the plane wave model without diffraction correction.

	P_r	P_t
τ_M (s)	7.312e-6 (0.003e-6)	7.2582e-6 (0.0007e-6)
τ_T (s)	—	1.30436e-5 (0.00002e-5)
Z_{WM}	2.131 (0.005)	2.079 (0.001)
α_1	10.4e-7 (0.1e-7)	7.72e-7 (0.05e-7)
α_2	1.21e-15 (0.03e-15)	8.2e-16 (0.1e-16)
β_1	9.9e-7 (0.2e-7)	7.41e-7 (0.04e-7)

TABLE IV: Estimated parameters and uncertainty (68% confidence) for plexiglass obtained with the SISO approach with diffraction correction.

	P_r	P_t
τ_M (s)	7.223e-6 (0.001e-6)	7.2213e-6 (0.0005e-6)
τ_T (s)	—	1.30477e-5 (0.00002e-5)
Z_{WM}	2.157 (0.005)	2.126 (0.001)
α_1	6.0e-7 (0.1e-7)	5.33e-7 (0.03e-7)
α_2	6.8e-16 (0.3e-16)	4.82e-16 (0.09e-16)
β_1	5.8e-7 (0.1e-7)	5.18e-7 (0.03e-7)