Theoretical and Computational Acoustics
D. Lee, Y-H Pao, M.H. Schultz, and Y-C Teng (Editors)
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Vibrational Response of a Finite Cylindrical Shell to Arbitrary Wave Incidence

Kenji Saijyou and Shigeru Yoshikawa
5th Research Center, Technical R&D Institute, Defense Agency
3-13-1 Nagase, Yokosuka, 239 Japan

Abstract

Vibrational responses of a finite submerged cylindrical shell induced by scattering of an arbitrary wave incidence are studied theoretically and experimentally. An analytical solution for the spherical wave incidence is extended by introducing the concepts of "insonified window" and "shape parameter". The measurement of surface vibration of a submerged, water-filled cylindrical shell induced by scattering is carried out using Scattering Generalized Near-field Acoustical Holography(SGENAH). The frequency of the incident wave is 3906Hz, 4883Hz, 5371Hz or 5859Hz. When the incident wave is 3906Hz, an almost uniform distribution of the surface vibration along the shell axis, which shows relatively higher amplitudes at front and rear sides to the projector, is obtained from our approximate theory. As the frequency goes up to 5859Hz, this almost uniform distribution is divided into three. Although the experimental results seem to be contaminated by the diffraction effects near the shell edge, a fairly good agreement between theoretical and experimental results assures the effectiveness of our approximate theory.

1 Introduction

Vibrational responses of a finite submerged cylindrical shell induced by scattering of an arbitrary wave incidence are studied theoretically and experimentally, where this *arbitrary* wave means a non-plane, non-spherical wave-front emanating from an actual transducer driven by a single frequency. Acoustic scattering of a cylindrical shell has been investigated by a large number of researchers because of its practical importance. Nevertheless, idealized assumptions (plane-wave incidence, infinite shell length, etc.) prevent us from analyzing realistic scattering problems[1, 2, 3, 4].

In order to remedy this circumstance, an analytical solution for the spherical wave incidence[4] is extended by introducing the concepts of "insonified window" and "shape parameter". To examine the validity of our approximate theory, calculated and experimental results are compared with each other. A fairly good agreement assures the effectiveness of our approximate theory.

2 Basic Physical Concepts

2.1 Shape parameter

An analytical solution of the scattering of a spherical wave incident on an elastic cylinder was derived by T.-B. Li and M. Ueda[4]. However, an actual transducer is far from a point source because of its geometry. The concept of "shape parameter" is defined in K-space as the difference in the shape of wave-front between the spherical wave and the wave from an actual transducer.

The vibrational response of an infinite shell induced by the scattering of the shperical wave incidence is generally described by the Bessel functions and scattering coefficient. When the scattering coefficient is multiplied by the shape parameter, the expression of the vibrational response is able to be extended for an arbitrary wave which is projected by an actual transducer. This shape parameter will be acquired by dividing the K-space representation of the incident wave from an actual projector by that of a spherical wave. More detailed derivation is described in the next section.

2.2 Insonified window

The concept of "insonified window" is a counterpart of the concept of "infinite baffle" in structural radiation problems. A radiating finite shell may be modeled as an infinite shell with an axially periodic distribution of vibration restricted to a finite part of interest. This modeling is justified if both ends of a finite cylindrical shell are simply supported[5, 6, 7]. For the scattering problems, a simply-supported finite shell seems to be approximated as an infinite shell whose insonified length is limited to the length of the shell. Namely, a simply-supported finite shell can be modeled as an infinite shell with a window whose aperture coresponds to the finite shell length. Therefore, the theory of the scattering from an infinite cylindrical shell[1] is applicable to the scattering problems on a finite cylindrical shell if the concept of this "insonified window" is properly introduced.

3 Theory

3.1 Derivation of the shape parameter

First, let us review the scattering of a spherical wave incident on an infinite cylindrical shell (see Fig. 1). A spherical wave with the frequency f is incident on the shell of the radius a. The sound source is located at $(d, \pi, 0)$ in the cylindrical coordinate system (r, θ, z). The acoustic pressure $p_i(r, \theta, z)$ of the spherical wave which illuminates the shell is generally expanded by the Bessel functions:

$$\begin{aligned} p_i(r,\theta,z) &= \frac{\exp(-jk\bar{r})}{\bar{r}} \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (-j)^{2n}\frac{j}{2} B_{|n|}(\sqrt{k^2-k_z^2}r) D_{|n|}(\sqrt{k^2-k_z^2}d) e^{jn\theta} e^{jk_z z} dk_z, \end{aligned} \quad (1)$$

where

$$B_{|n|}(\sqrt{k^2-k_z^2}r) = \begin{cases} J_{|n|}(\sqrt{k^2-k_z^2}r) & \text{if } k > k_z, \\ I_{|n|}(\sqrt{k_z^2-k^2}r) & \text{if } k < k_z, \end{cases}$$

and

$$D_{|n|}(\sqrt{k^2-k_z^2}r) = \begin{cases} H_{|n|}^{(2)}(\sqrt{k^2-k_z^2}r) & \text{if } k > k_z. \\ K_{|n|}(\sqrt{k_z^2-k^2}r) & \text{if } k < k_z. \end{cases}$$

In Eq.(1) $\bar{r}$ denotes the distance between the sound source $(d, \pi, 0)$ and the field point (r, θ, z). Functions J_n and $H_n^{(2)}$ are respectively the Bessel function of the first kind and the Hankel function of the second kind, where n is the order. Functions I_n and K_n are the modified Bessel functions of the first and second kind, respectively. The k is the wavenumber of the radiation wave and k_z the z directional component of the wavenumber. The radial vibrational response $v_r(a, \theta, z)$ induced by the scattering of the shperical wave incidence is given as follows:

$$\begin{aligned} v_r(a,\theta,z) &= \frac{1}{j\rho ck} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sqrt{k^2-k_z^2}(-j)^{2n}\frac{j}{2} \\ &\times D'_{|n|}(\sqrt{k^2-k_z^2}r') D_{|n|}(\sqrt{k^2-k_z^2}d) g_n(k_z) e^{jn\theta} e^{jk_z z} dk_z|_{r'=a}, \end{aligned} \quad (2)$$

where $g_n(k_z)$ is the scattering coefficient[9] and the prime denotes the differentation with respect to the argument. The ρ is the density of the fluid and c the sound speed of the fluid.

Next, let us extend the incident wave from the spherical one to arbitrary one. Assume that an actual transducer is constructed by the infinitesimal discrete point sources of respective amplitude $p_d(r_0, \theta_0, z_0)$ at the point (r_0, θ_0, z_0). If the transducer is located outside the cylinder of radius $r_1 (> a)$, the arbitrary incident wave pressure can be expressed as follows:

$$\begin{aligned} p_{ia}(r,\theta,z) &= \int_{r_1}^{\infty} dr_0 \int_0^{2\pi} d\theta_0 \int_{-\infty}^{\infty} dz_0 \; p_d(r_0, \pi-\theta_0, z_0) \frac{\exp(-jk\bar{r}_0)}{\bar{r}_0} \\ &= \int_{r_1}^{\infty} dr_0 \int_0^{2\pi} d\theta_0 \int_{-\infty}^{\infty} dz_0 \; p_d(r_0, \pi-\theta_0, z_0) \\ &\times \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (-j)^{2n}\frac{j}{2} B_{|n|}(\sqrt{k^2-k_z^2}r) D_{|n|}(\sqrt{k^2-k_z^2}r_0) e^{jn(\theta-\theta_0)} e^{jk_z(z-z_0)} dk_z \\ &= \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} (-j)^{2n}\frac{j}{2} B_{|n|}(\sqrt{k^2-k_z^2}r) D_{|n|}(\sqrt{k^2-k_z^2}d) \alpha_n(k_z) e^{jn\theta} e^{jk_z z} dk_z, \end{aligned} \quad (3)$$

The "shape parameter" $\alpha_n(k_z)$ in Eq.(3) is given by comparing the second and third expression in the right side of Eq.(3) as follows:

$$\begin{aligned}\alpha_n(k_z) &= \frac{\int_{r_1}^{\infty} dr_0 \int_0^{2\pi} d\theta_0 \int_{-\infty}^{\infty} dz_0\, D_{|n|}(\sqrt{k^2-k_z^2}r_0)\exp(-jk_z z_0)\exp(-jn\theta_0)}{D_{|n|}(\sqrt{k^2-k_z^2}d)} \\ &= \frac{F[p_{ia}(r,\theta,z)]}{(-j)^{2n}\frac{j}{2}D_{|n|}(\sqrt{k^2-k_z^2}d)B_{|n|}(\sqrt{k^2-k_z^2}r)},\end{aligned} \tag{4}$$

where F denotes the 2-D spatial FFT operetor which is defined as

$$F[p_{ia}(r,\theta,z)] \equiv (\frac{1}{2\pi})^2 \int_0^{2\pi} d\theta \int_{-\infty}^{\infty} dz\; p_{ia}(r,\theta,z)\exp(-jn\theta)\exp(-jk_z z), \tag{5}$$

The multiplication of the scattering coefficient $g_n(k_z)$ by this shape parameter $\alpha_n(k_z)$ extends Eq.(2) on the spherical wave incidence to the equation on an arbitrary wave incidence:

$$\begin{aligned}v_r(a,\theta,z) &= \frac{1}{j\rho ck}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sqrt{k^2-k_z^2}(-j)^{2n}\frac{j}{2} \\ &\times\; D'_{|n|}(\sqrt{k^2-k_z^2}r')D_{|n|}(\sqrt{k^2-k_z^2}d)\alpha_n(k_z)g_n(k_z)e^{jn\theta}e^{jk_z z}dk_z|_{r'=a},\end{aligned} \tag{6}$$

Substitution of Eq.(4) into Eq.(6) brings

$$\begin{aligned}v_r(a,\theta,z) &= \frac{1}{j\rho ck}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sqrt{k^2-k_z^2} \\ &\times\; D'_{|n|}(\sqrt{k^2-k_z^2}r')\frac{F[p_{ia}(r,\theta,z)]}{B_{|n|}(\sqrt{k^2-k_z^2}r)}g_n(k_z)e^{jn\theta}e^{jk_z z}dk_z|_{r'=a},\end{aligned} \tag{7}$$

Equation (7) tells us that the experimentally-obtained information of $p_{ia}(r,\theta,z)$ is necessary and sufficient to calculate $v_r(a,\theta,z)$. Also, the shape parameter does not explicitly appear in Eq.(7). The essential point is that the shape parameter is not a function of the field variables r, θ, z, but a function of k_z only as known from Eq. (4).

3.2 Derivation of the insonified window

In this subsection, we attempt to introduce the finiteness of shell length into our theory by analogy with structural radiation. A finite shell whose both ends are simply supported is closely approximated as an infinite shell whose corresponding finite portion is enclosed by the "infinite baffle" [5, 6, 7] [see Fig.2(a)]. Since the distribution of vibration on the finite portion is spatially periodic, the z directionl component of the wavenumber is restricted to

$$k_z = \frac{m\pi}{L}, \tag{8}$$

where the axial mode number m takes integer and L denotes the shell length.

In the scattering problems, a simply-supported finite shell may be modeled as an infinite shell whose insonified length is limited to the length of the shell. The influence of the incident wave comes in through the "insonified window", which is expressed as

$$W(z) = \begin{cases} 1 & \text{if } |z| \le L/2, \\ 0 & \text{otherwize,} \end{cases} \tag{9}$$

By analogy with the infinite baffle, the outer region of the insonified window is supposed not to be moved by the acoustic wave coming through the window. Since $W(z)$ is applied to the incident wave, Eq.(7) is generalized as

$$\begin{aligned} v_r(a,\theta,z) &= \frac{1}{j\rho ck}\int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty}\sqrt{k^2-k_z^2} \\ &\times D'_{|n|}(\sqrt{k^2-k_z^2}r')\frac{F[p_{ia}(r,\theta,z)W(z)]}{B_{|n|}(\sqrt{k^2-k_z^2}r)}g_n(k_z)e^{jn\theta}e^{jk_z z}dk_z|_{r'=a}, \end{aligned} \tag{10}$$

Note that k_z in Eq. (10) must satisfy Eq. (8). The effects of realistic source geometry and shell length finiteness are respectively incorporated in Eq.(10) through $p_{ia}(r,\theta,z)$ and $W(z)$.

4 Comparison Between Theoretical and Experimental Results

The experimental results on a finite cylindrical shell are obtained by Scattering Generalized Near-field Acoustical Holography (SGENAH)[8]. Two kinds of measurement should be executed to derive the scattered hologram. First, the pressure on the two-dimensional measurement contour (called "hologram surface") is acquired with the shell. These pressure data (called "superimposed hologram") consist of the incident and scattered waves. Second, the pressure data on the same hologram surface are obtained without the shell. Such pressure data (called "incident hologram") include the incident wave only. The incident hologram data are also used for the calculation of Eq. (10). The "scattered hologram", which is obtained by subtracting the incident hologram from the superimposed hologram, is transformed to the shell surface vibration by GENAH signal processing. See Ref. [8] for more details on the Scattering GENAH signal processing.

An experimental cylindrical shell is made of SUS ♯ 304 stainless steel. The shell is 800 mm in length, 208 mm in inner diameter, and 3.0 mm in thickness. It is sealed with endcups and rubber packings to approximate the boundary condition of simple support. Note that the shell is filled with water. The sound velocity in water is 1455.3m/s. The radius of the measurement contour is 120 mm. The transducer is located at point $(r,\theta,z) = (d,\pi,0)$, here d=300 mm (cf. Fig.1), and the transmitted frequency is 3906Hz, 4883Hz, 5371Hz or 5859Hz (ka=1.75, 2.19, 2.41 or 2.63).

Figure 3 represents the calculated and measured amplitude of surface vibration induced by scattering. The logarithmic level of the surface velocity $20\log|V_r(z,\theta)|$, where V_r is normalized by the maximum value, is drawn in Fig.3. In this figure, the horizontal axis indicates the circumferential angle θ and the vertical axis the shell axis z. Figure 3(a), where the frequency of the incident wave is 3906Hz, shows higher amplitudes near $\theta = 0$ (rear side to the projector) and $\theta = \pi$ (front side to the projecter). The corresponding experimental result shown in Fig. 3(e) exhibits similar distribution of the velocity amplitude except for streaks of lower amplitudes around 100 mm from the shell edge. Figures 3(b) and 3(f) for 4883Hz show the distribution similar to Figs. 3(a) and 3(e), respectively. When the frequency goes up to 5371Hz, an almost uniform distribution along the shell axis shown in Figs. 3(e) and 3(f) is divided into three as shown in Fig. 3(g). This theoretical tendency is well reflected in Fig. 3(c). A higher frequency of 5859Hz reveals this tendency more clearly as shown in Figs. 3(d) and 3(h). Although the experimental results of Fig. 3(a) to 3(d) seems to be contaminated by the diffraction effects near the shell edge, a fairly good agreement between theoretical and experimental results is obtained.

5 Conclusions

The vibrational responses of a finite submerged cylindrical shell induced by the scattering of an arbitrary wave incidence are studied theoretically and experimentally. The concepts of "shape parameter" and "insonified window" are introduced to create an approximate theory. The former concept defines the difference in the shape of wave-front between the spherical wave and the actual measured incident wave. The latter concept forms the theoretical basis to model a simply-supported finite shell as an infinite shell with a transparent window of finite shell length through which the incident wave is addmitted.

The validity of our theory is demonstrated by the comparison between theoretical and experimental results of the shell surface vibration. For specific display a submerged water-filled shell is employed. The frequency is restricted to 3906 to 5859 Hz. The experimental results is acquired by Scattering GENAH[8]. An almost uniform distribution along the shell axis in lower frequencies tends to be divided into three in higher frequencies.

The measured results seem to be contaminated by the diffraction effects near the shell edge. Nevertheless, a fairly good agreement between theoretical and experimental results assures the effectiveness of our approximate theory.

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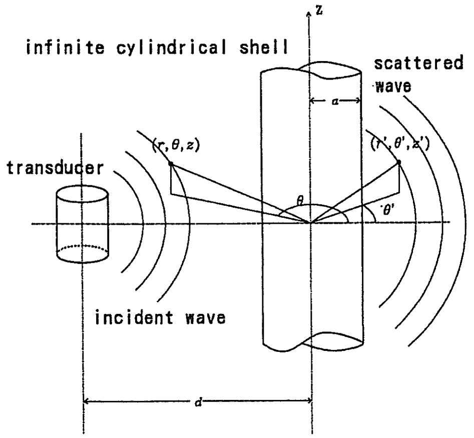

Figure 1: Configuration and coordinate system for the scattering by an infinite cylndrical shell.

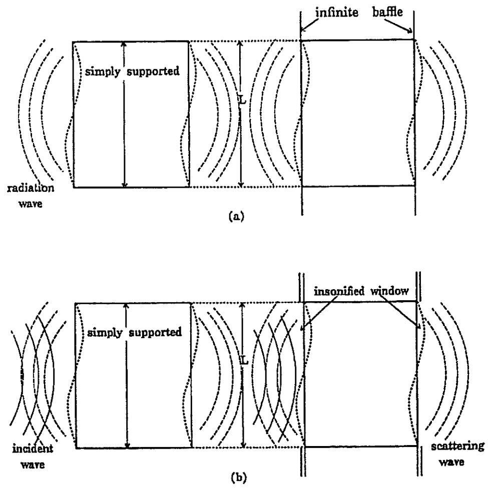

Figure 2: "Insonified window" concept and "infinite baffle" concept. (a)Infinite baffle in radiation problems. The dashed line indicates the radiation wave.
(b) Insonified window in scattering problems.
A simply-supported finite shell on the left is approximated by an conceptualized infinite shell on the right.

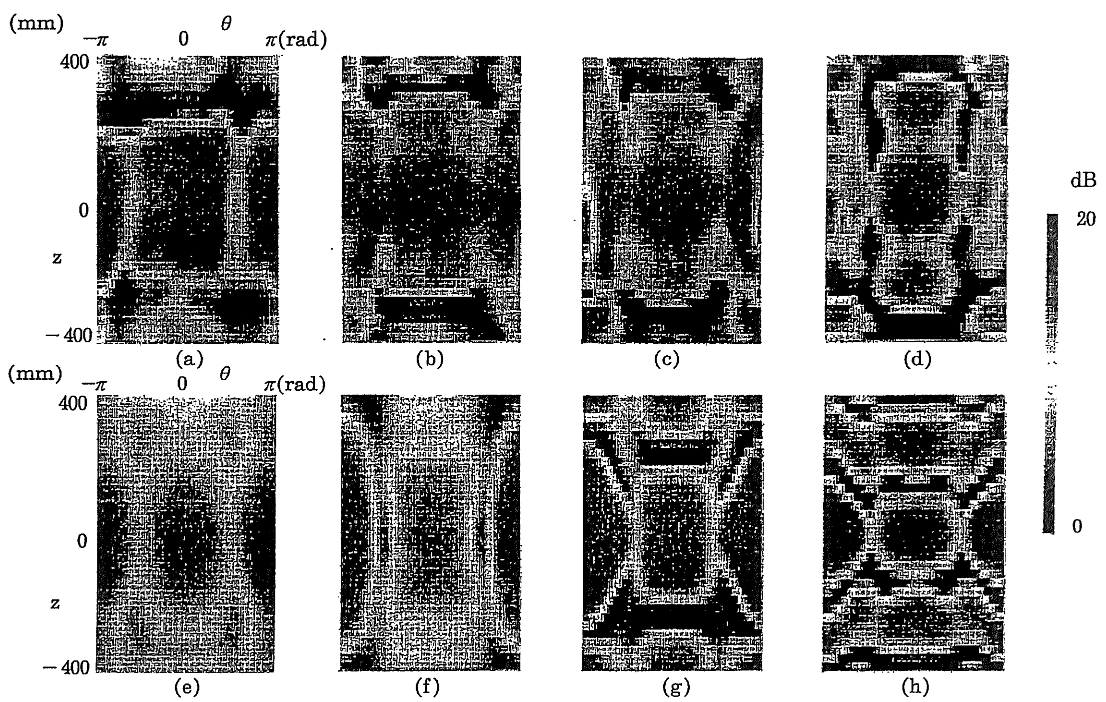

Figure 3: Surface-velocity amplitude induced by the scattering pressure. The upper row shows the measured result, and the lower row the calculated result. (a) and (e):3906Hz; (b) and (f):4883Hz; (c) and (g):5371Hz; (d) and (h):5859Hz.