

## The Constrained Parameter Inversion of Seismic Data Using the Simulated Annealing Algorithm

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### Abstract

Seismic parametric inversion based on optimization method is referred to a nonconvex optimization problem. The simulated annealing (SA) method is adopted in this paper. SA method can obtain the global optimum inversion solution, it is applicable to nonconvex optimization problem of seismic parametric inversion. This new inversion method has advantage over conventional simulated annealing method in computational speed and precision because it uses the constrained condition for controlling inversion process. This method advance the applicability of simulated annealing to real seismic data processing. The examples of synthetic and real data show that the accuracy of the method is satisfactory.

**Key words:** Simulated annealing, seismic inversion, constrained condition, optimization.

## INTRODUCTION

Seismic inversion can obtain the information on the property of lithology from seismic data, it is the fundamental technique of predicting the oil-gas reservoir. In the research of seismic inversion, many methods involve finding the best model which would produce the observed data, the optimum model is that model for which misfit function (e.g. error energy function) is a minimum. This kind of inversion actually is to solve the optimization problems. Most seismic parametric inversions obtain solution of inversion problem by using optimization method. For many years, seismic inversion usually adopts the traditional optimization method (steepest descent method, DFP method etc.), however, the seismic inversion problem is nonconvex optimization problem, the misfit function has many extrema, it is difficult to obtain the global optimum solution of inversion by using traditional optimization method (Mora, 1987; Pica et al., 1990).

Simulated annealing (SA) method originally is the analog to a physical process in which a solid heated by increasing the temperature, followed by slow cooling until it reaches the minimum energy state where it forms a crystal (Metropolis et al., 1953). SA method has been applied to solving optimization problem (Kirkpatrick, 1983). Seismic inversion can converge to a global optimum solution by using SA method, and keep the inversion from being caught in a local optimum answer. Since the work of Rothman (1985, 1986), conventional simulated annealing (CSA) has been used in seismic statics correction problems. The cooling scheme of CSA method is very slow, so CSA method is too expensive. Broadband constrained inversion (BCI) is a new seismic inversion method developed in recent years (Martinez et al., 1989; Mansfield, 1990; Zhou et al., 1993). BCI method simultaneously makes use of an initial interpretation model by prior information, seismic data, and geophysical well-log data to obtain the solution of seismic inversion. Although BCI method can enhance the vertical resolution of inver-

sion solution, it can only solve the convex optimization problem for using traditional optimization method.

We combine the seismic parametric inversion using simulated annealing method and the constrained condition inversion. The constrained condition is imposed by the prior well-log, seismic, and geological information. The constrained inversion using SA can not only solve the problem of searching global optimum but also speed up the convergence of simulated annealing inversion by using constrained condition, therefore this inversion method is more reasonable and effective.

In this paper, the examples of synthetic are given. The theoretical trial shows this inversion method is feasible for computation of complicated geological structure. The result of processing real data proves this method is practical, it advances the applicability of simulated annealing inversion method to the stage of processing real seismic data.

## ALGORITHM

### 1. Simulated annealing

The computational process of simulated annealing inversion is to realize a random walk in the model space. The probability density function is given by

$$P(m) = \frac{e^{-\frac{S(m)}{T}}}{D}, \quad m \in \Omega \quad (1)$$

where  $m$  is the parameter model,  $S(m)$  is misfit function,  $T$  is the control parameter (temperature),  $\Omega$  is the model space, and  $D$  is given by

$$D = \sum_{m \in \Omega} e^{-\frac{S(m)}{T}} \quad (2)$$

Simulated annealing inversion actually generates a random process under the

control of parameter  $T$ . In iterative process of inversion, traditional optimization method only accept those models whose misfit function was descent. Let us consider a acceptance function

$$\alpha(\Delta S, T) = \begin{cases} 1 & , \Delta S < 0 \\ e^{-\frac{\Delta S}{T}} & , \Delta S > 0 \end{cases} \quad (3)$$

where  $\Delta S$  is the change of misfit function. At each step of SA inversion, SA method evaluates the change of the misfit function by the acceptance function. The model is accepted if  $\Delta S < 0$ . If  $\Delta S > 0$ , judges whether the model is accepted or not by the acceptance function of the model. Because SA method will accept the model whose misfit function rose (i.e.  $\Delta S > 0$ ), SA method can avoid being trapped in local extrema.

## 2. Misfit function

The solution of the forward problem was written as

$$y = f(m) \quad (4)$$

Let  $y_0$  denote the observed data,  $n$  denote the difference between the forward modeling  $f(m)$  and the observed data  $y_0$ ,  $n$  defined as

$$n = y_0 - y \quad (5)$$

Using Bayesian estimation theory(Bard, 1974), we obtain the misfit function as

$$S(m) = \frac{1}{2} \{ [y_0 - f(m)]^T C_n^{-1} [y_0 - f(m)] + (m - m_0)^T C_m^{-1} (m - m_0) \} \quad (6)$$

This is a weighted least-squares norm. In equation (6),  $C_n^{-1}$  and  $C_m^{-1}$  are the inverse covariance matrix of data and model,  $m_0$  is the initial model.

### 3. Constrained condition

Constrained condition includes the range of model parameters and the initial model. The initial model can be obtained through interpretation of seismic data and well-log data. It is important to restrict the range of searching parameters. Discrete parameter model defined as

$$m = \{m_i\}, \quad i = 1, 2, \dots, N \quad (7)$$

Let  $m_i^{up}$ ,  $m_i^{inf}$  denote superior limit and inferior limit of parameter  $m_i$ , constrained condition given by

$$m_i^{inf} \leq m_i \leq m_i^{up} \quad (8)$$

The parameter model space is written as

$$\Omega = \{m \mid m_i^{inf} \leq m_i \leq m_i^{up}, \quad i = 1, 2, \dots, N\} \quad (9)$$

We can determine the space  $\Omega$  with reference to prior seismic data and well-log data.

## RESULTS

### 1. Examples of synthetic data

In this paper, the examples of synthetic data computed by using the constrained SA method are presented. A flat-layer model is shown in Figure 1(a), velocity value of model is in the range of 2300 to 4100 m/s. Figure 2(a) and Figure 6(a) are noise-free and noise synthetic seismic data with the flat-layer model respectively. We use the one-dimensional acoustic approximation and the finite difference method for forward calculation. Source function is a Ricker wavelet (Ricker, 1953) with a main frequency of 70 Hz.

The match of true model and result of inversion is good (Figure 1(c)), error energy less than 1.0 percent (Figure 1(b)). There is excellent agreement between the theory seismic data and synthetic data (Figure 2).

The solutions of using a loose constrained condition and a simplified initial model are shown in Figure 3 and Figure 4. The loose constrained condition and the simplified initial model will slow down the convergence of simulated annealing inversion. SA constrained inversion using prior constrained condition can save computer time and improve the efficiency of simulated annealing algorithm.

Figure 5 and Figure 6 are the presentation of our method using noise data. Although the result gives rise to errors (Figure 5(a)), the match of true model and result of inversion is good (Figure 5(b)), and the synthetic data of inversion is in good agreement with noise data (Figure 6).

### 2. Example of real data

We use our inversion method for processing real seismic data collected in Daqing of China. Figure 7 shows velocity and density of inversion (dashed line) are in agreement with real well-log data (solid line). The frequency bandwidth of inversion is extended as compared with real seismic data (Figure 8). Figure 9 is profiles of velocity and density. There is a zone of high velocity and low density on the profiles, at 1.55 sec. and CDP 310~350. The zone is an oil-sand layer. Figure

10 is a comparison between real seismic data and synthetic data of inversion. The match between the real data and synthetic data is good.

## CONCLUSION

We have discussed constrained inversion using simulated annealing, and applied the method to the parametric inversion of synthetic and real seismic data. The results of our method indicate: (1) It is possible that simulated annealing method can be used in seismic parametric inversion. The solution of SA constrained inversion method is correct and reasonable. The frequency bandwidth of the solution is broad; (2) If constrained condition is reasonable, SA constrained inversion is able to speed up convergence of inversion process, save computer time and improve the efficiency of simulated annealing algorithm; (3) Our inversion method has ability of noise resistance. The result of real data processing is satisfactory.

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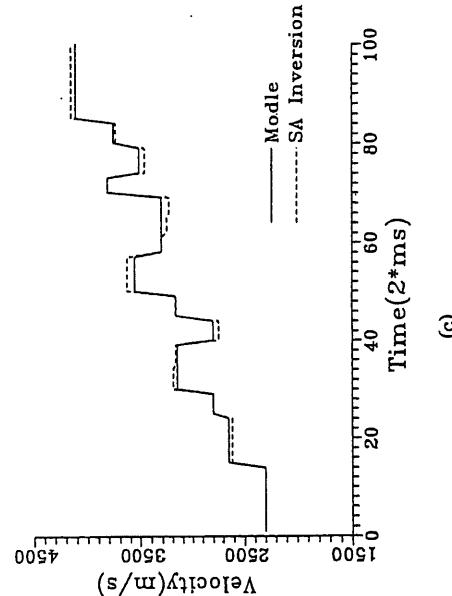
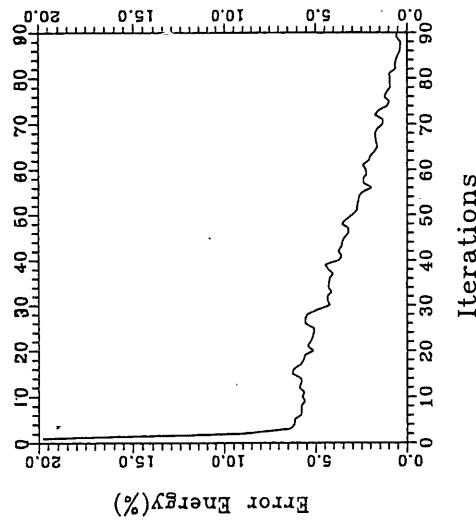
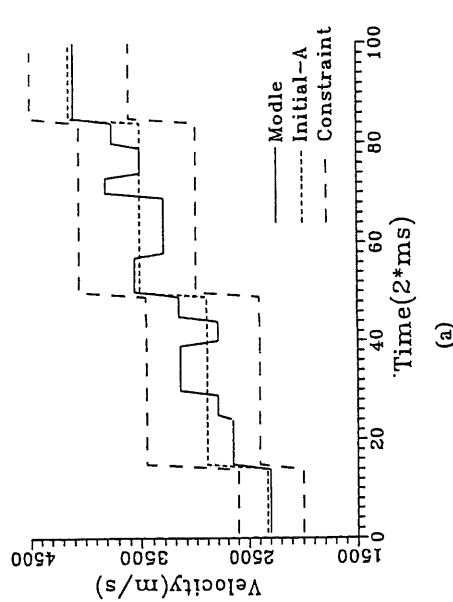


Fig. 1 (a) The flat-layers model, initial model and restrictive condition;  
 (b) Error energy function (misfit function);  
 (c) Comparison between the true model (solid line) and the result of inversion (dashed line).

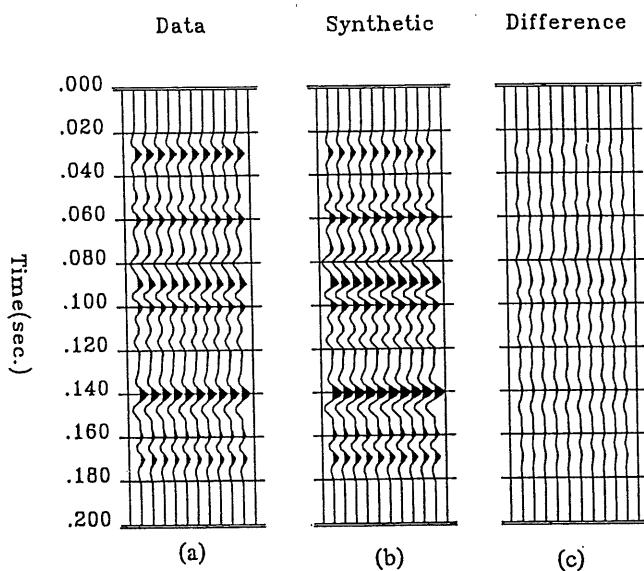
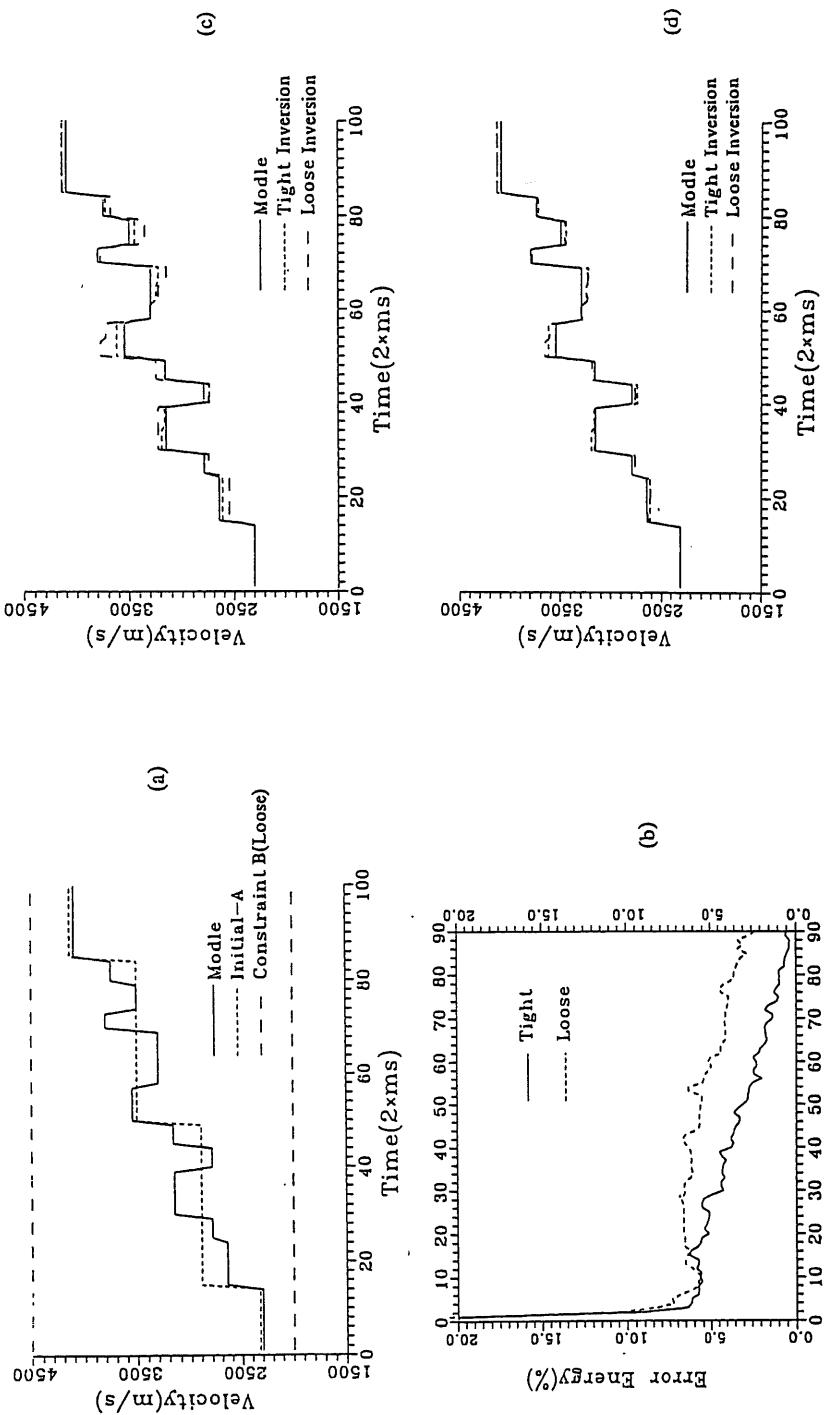


Fig. 2 (a) Theory seismic noise—free data;  
(b) Synthetic data computed by the inversion;  
(c) Difference between theory data and synthetic  
data.

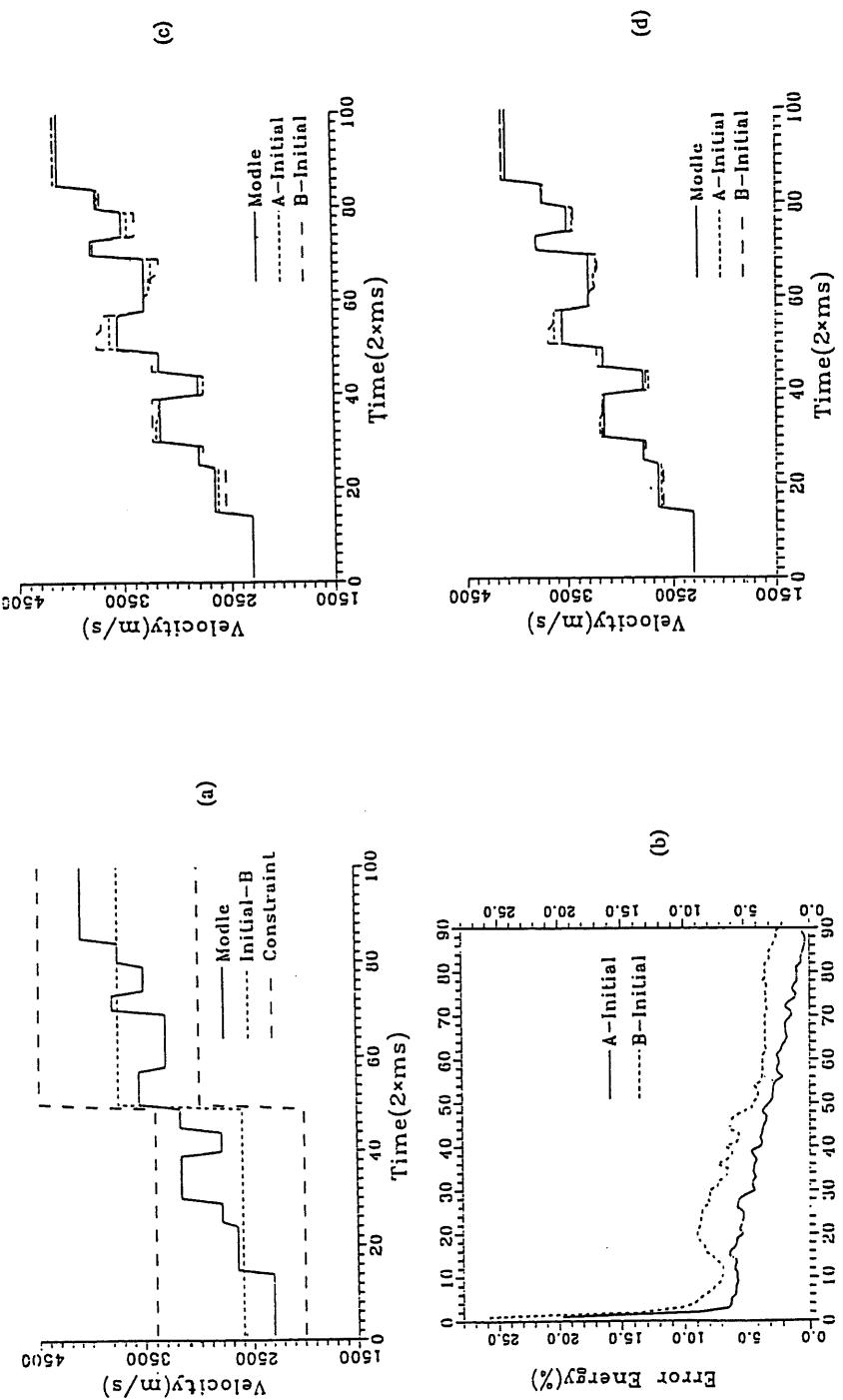


### Iterations

(a) Theory model, initial model and a loose restrictive condition;

(b) Error energy function (misfit function);

(c), (d) The influence of tight and loose restriction upon inversion, (c) is results of the inversion at iteration 70, and (d) is the results of inversion at iteration 90.



### Iterations

Fig. 4 (a) Theory model, initial model and a simplified initial model;  
(b) Error energy function (misfit function);

(c) , (d) The influence of A initial model(Fig. 3 (a)) and B initial model(simplified initial model) upon inversion, (c) is results of the inversion at iteration 70, and (d) is the results of inversion at iteration 90.

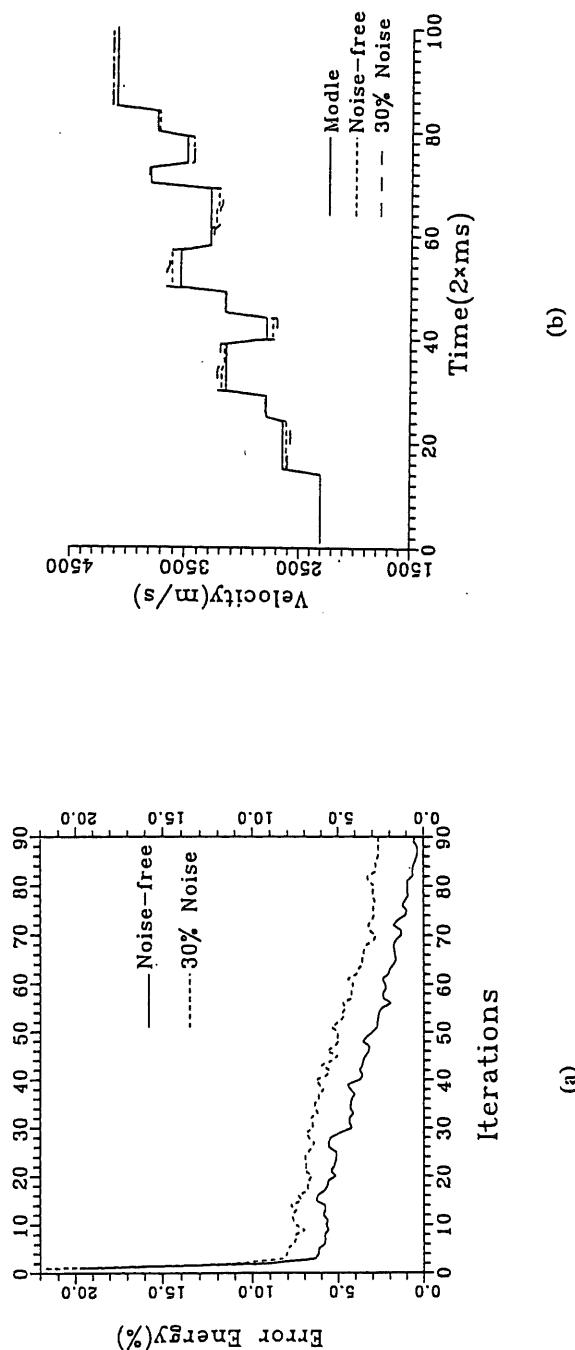


Fig. 5 (a) Error energy function (misfit function);  
 (b) Comparison results of the inversion between  
 noise-free data and noise data.

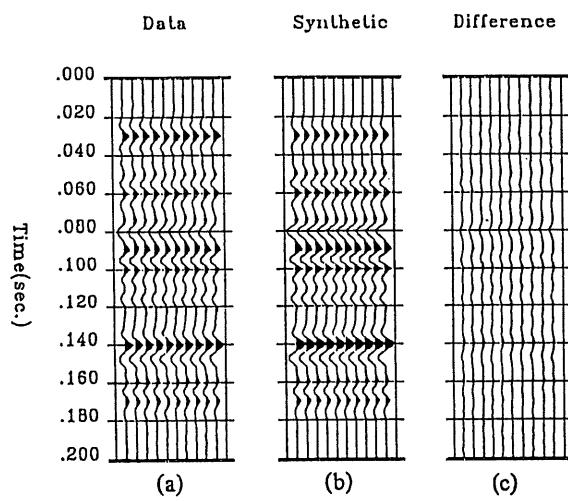


Fig. 6 (a) Theory seismic data for 30 percent random noise;  
(b) Synthetic data computed by the inversion;  
(c) Difference between theory data and synthetic data.

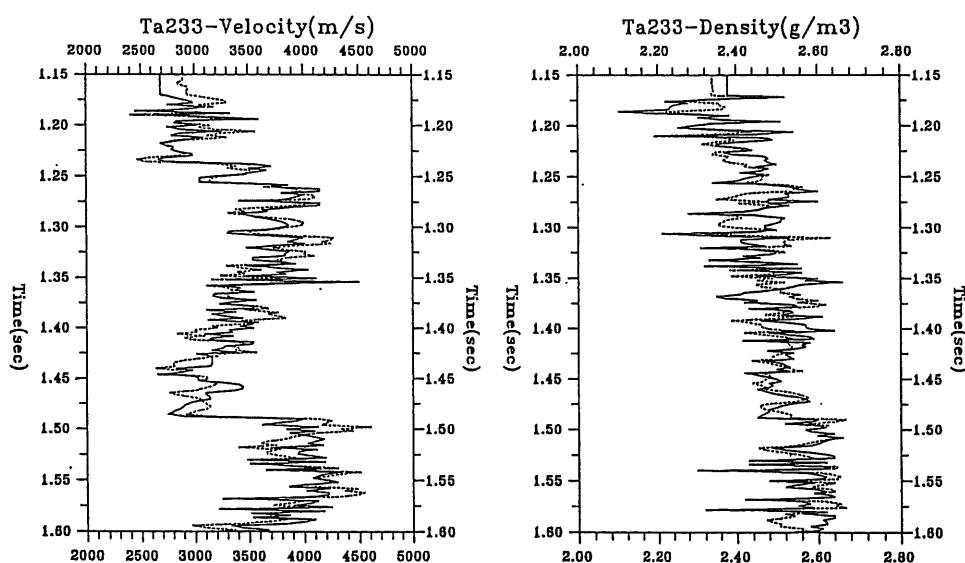


Fig. 7 The comparison between well-log data(solid line) and result of inversion(dashed line).

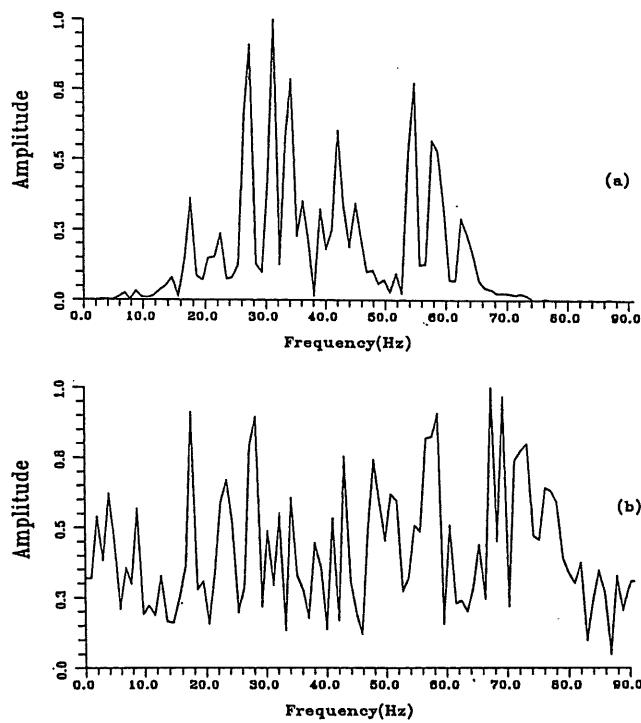


Fig. 8 (a) The frequency spectrum of real data;  
(b) The frequency spectrum of inversion data.

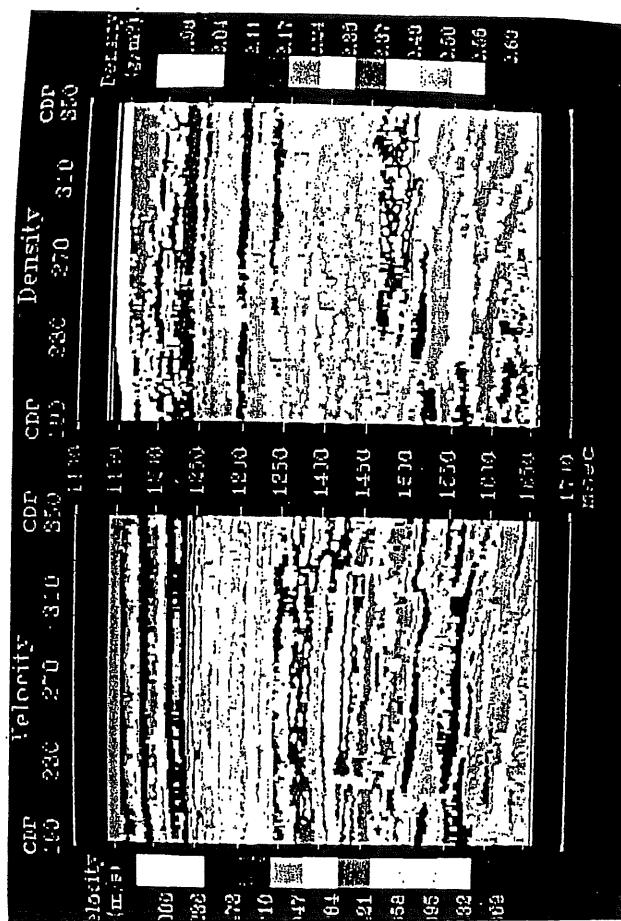


Fig. 9 Velocity and density section of the inversion

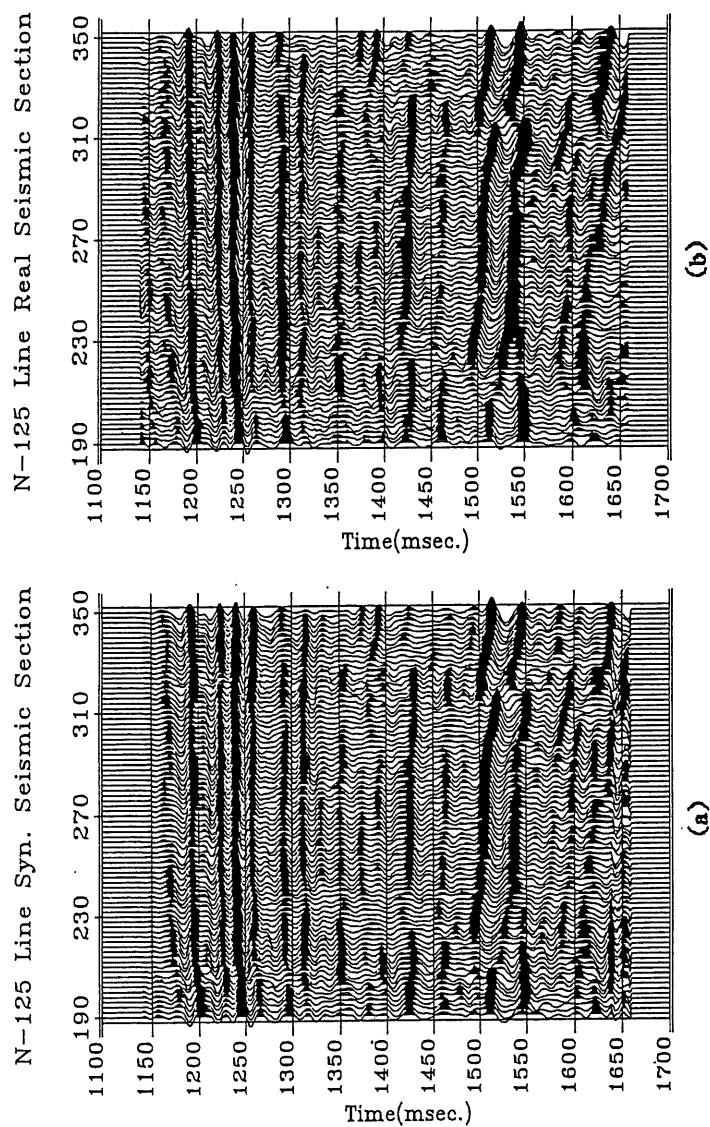


Fig. 10 The profiles of seismic data  
(a) Synthetic seismic data of the inversion.  
(b) Real seismic data;