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Scattering Kernel Approach to Bottom Loss and Scattering in a Parabolic Equation Model

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Abstract

We investigate the inclusion of bottom scattering and loss in a PE propagation model by using a scattering kernel approach which has previously been applied at the ocean surface. Results are presented in which the scattering angular distribution is generated from Lambert's law.

1 Introduction

In this communication we discuss the implementation of what has sometimes been called the "scattering kernel" approach to treating scattering from a rough interface in the modeling of acoustic propagation in the ocean using a parabolic equation (PE) approximation. There are other approaches, such as the conformal mapping technique of Dozier[1] and the "Monte Carlo" or Brute-force approach which I have described earlier [2].

The "scattering kernel" method in the form offered by Dozier, et al [3] and Schneider [4], is a generalization of a method introduced by Moore-Head, et al [5], which treated only surface loss, that is, did not attempt to incorporate rough surface scattering. That method involved analyzing the acoustic field at the ocean surface in vertical wave-number space, applying an appropriate loss function to the acoustic field in a layer adjacent to the surface, and then transforming to real space. Dozier's generalization is the more elaborate of the two, incorporating as it does a scheme for guaranteeing energy conservation as the PE field is modified. In every case thus far, the application has been to the ocean surface.

As is well known, there are serious objections to this approach, objections which have been discussed in various places, including the paper by Schneider. The modification of the PE field "on the fly" seriously compromises the numerical integrity of the solution method, and creates a situation in which numerical instability or divergence is likely to occur. In practice, the work of Moore-Head, et al, Dozier, et al, and Schneider, suggest that with care, the method can produce both stable and useful results. Briefly, and superficially, the requirement is that the changes in the complex pressure field due to scattering or loss mechanisms, per range step, be small. In practice, this may mean that there are real physical limitations on the magnitude of the scattering or loss which can be handled using this method. This we shall explore below.

In this work, we apply the scattering kernel method to scattering from an acoustically rough ocean bottom. The scattering kernel is obtained from one of several possible approaches, including, as in the work of Dozier, et al, using a model of surface roughness obtained from a deterministic model of cylindrical bosses [3] Other methods include the very simplest, which is to generate diffuse scattering from Lambert's law, or using the method of moments to generate a scattering angular distribution. I have discussed other possibilities elsewhere[6]. The details of the method of obtaining these angular distributions are of secondary interest at this point; of much more immediate interest is the question of applicability of this method to the ocean bottom and its viability numerically.

Bottom loss is introduced here as a test of the effectiveness and stability of the method, and only incidentally as a representation of processes which are otherwise neglected. To first order, at least, bottom loss is already accounted for by attenuation introduced in the sediment.

2 Calculations and Numerical Experiments

Using a finite-element PE code originally written by Collins [7], we have implemented a version of the scattering kernel method which modifies the transformed pressure field in a layer adjacent to the bottom, using a scattering matrix S which operates on the pressure field vector ψ in that layer. As indicated above, the scattering matrix may be obtained in a variety of ways, and indeed, may be constructed in such a way as to scatter energy from one angular range to another, rather than from a realistic model of rough surface scattering. In the present implementation, the field is analyzed by performing an FFT to vertical wave-number space, but it is reasonable to assume that the spectral decomposition approach [8] could also be used. In what follows, we will concentrate solely on the numerical questions which arise, rather than on comparison of results with benchmark solutions or real environments.

In the present studies, three different approaches were used in investigating the sensitivity of the marching solution to modifications of the pressure field: 1) energy was redirected from one angular range to another by tailoring the scattering matrix to that specific purpose, 2) energy was extracted from the field in the sediment and returned to the water column, and 3) energy which would be specularly scattered by the PE model was scattered diffusely according to some prescription (e.g., Lambert). Obviously only the latter approach has any meaning physically, but the other two demonstrate the efficacy of the program in modifying the pressure field to produce an arbitrary scattering result. In addition to the scattering computations, bottom loss was implemented, in a fashion similar to that of Moore-Head, et al, or Dozier, et al, and the same numerical issues arise in this context as well. In these calculations there are two senses in which the size of the FFT window in depth is important. On the one hand there is the competition between employing a small FFT depth window, which localizes the analyzed field near the bottom, and the angular resolution provided by a larger window. On the other, there is also tension between the desire to enlarge the FFT window in order to have the desired effect on the pressure field (i.e., the desired scattering), and the need to insure a small change in the field, per range step. In the end there is about as much art as science involved in these decisions, and certainly the method seems ill-suited to an automated mode of operation.

Table 1: Acoustic Parameters. The frequency is 25 or 75 Hz

$$c_o = 1500msec^{-1}$$
$$\alpha_w = 0.0$$
$$\alpha_{sed}(3000) = 0.5$$
$$\alpha_{sed}(5000) = 10.0$$
$$npade' = 2$$
$$Z_S = 150m$$

In what follows, we show examples of calculations for a single acoustic environment, whose details are given in Table I. The diffuse scattering of the reflected field is accomplished using a Lambert's Law expression of the form

$$S(k_1, k_2) = \mu sin(\theta_1) sin(\theta_2), \tag{1}$$

where θ_1 and θ_2 are the incident and scattered grazing angles. (k_1 and k_2 are the incident and final vertical wavenumbers, given by $k_o sin\theta$ The primary justification is ease of implementation. The computation essentially consists of calculating ψ' using:

$$\psi' = FFT^{-1}[S(k_1, k_2)FFT(\psi)] \tag{2}$$

although smoothing of the pressure field near the bottom is desirable to reduce the possibility of aliasing [5], [4].

Calculations which have been performed to this point have generally been carried out with acoustic environments which produced strong bottom interaction, for obvious reasons. Examples are a shallow water environment, or a deeper one with downward-refracting sound speed profiler; frequencies used ranged from 25 to 125 Hz. The number of points in the FFT window was chosen to be 16 or 32, i.e., 16 or 32 z-mesh points, typically 90-180 m in a deep water environment, 4-8 m in shallow water.

3 Results of Computations

Unlike the brute force approach, which ensemble-averages the results of many propagation runs for statistically equivalent randomly rough surfaces, the scattering kernel method puts only modest demands on computational resources; typical execution times are 100 seconds for 5000 range steps on an IBM RS6000 model 365, obviously depending on the number of points in the FFT window. The critical computational problem is the one mentioned earlier, of numerical stability. It should not come as a surprsise that if the complex field is changed markedly at each range step, the field very quickly diverges. The results given below show examples of this divergence.

The results of calculations with no scattering, scattering alone, and both scattering and bottom loss are shown below, for the acoustic environment given in Table I. The results are generally typical of the spectrum of calculations done for other environments. For comparison, Figure 1 shows a propagation run with no loss or scattering. In vertical wave-number space, at a distance of 5000m from the source, the field in the scattering layer is as shown in Figure 2, in which the incident and specularly reflected energies are seen. If bottom scattering is turned on (using Lambert's Law), the pressure field beyond about 15Km is modified as energy is scattered diffusely out of the specular direction (Figure 3). The Lambert's Law parameter μ has the value μ=0.002 in Figure 3, which represents a scattering strength of -27 dB. The effect of the scattering has been to spread the energy from the specular directions, reducing the intensity beyond about 25Km. Figure 4 shows the pressure field when a bottom loss of α =10 dB per bounce has been introduced, and Figure 5 compares this field with the $\alpha = \mu = 0$ case of Fig. 1, by subtracting the intensities. In this case, both the field beyond about 25 Km and the field in the bottom are reduced. Finally, Figure 6 exhibits the transmission loss with and without scattering, and with and without loss, for a receiver at 150m.

The problems that arise are illustrated in Figures 7-8. In the former with μ=.005 (-23 dB), the field diverges rapidly after 13 km. This is also seen dramatically in the field plot in Figure 8. It is clear then, that in this range-independent environment in which the same bottom roughness obtains over 20 Km, only weak scattering can be aplied if the propagation code is to yield meaningful results.

4 Conclusion

While admittedly these results are preliminary and we have adopted a very simple form for the scattering cross section, they shed some light on the the problems inherent in the method. As expected, the main limitation on the method is provided by the need to keep the change in the complex pressure field, per range step, small. It might be noted that the use of the methof of Dozier, et al, while it increases the computational load (a matrix diagonalization is required at each range step in a range-dependent environment), seems to enhance the stability of the scattering kernel technique, permitting a somewhat wider range of scattering strengths to be employed. On the other hand, it does not, per se, eliminate the numerical problems.

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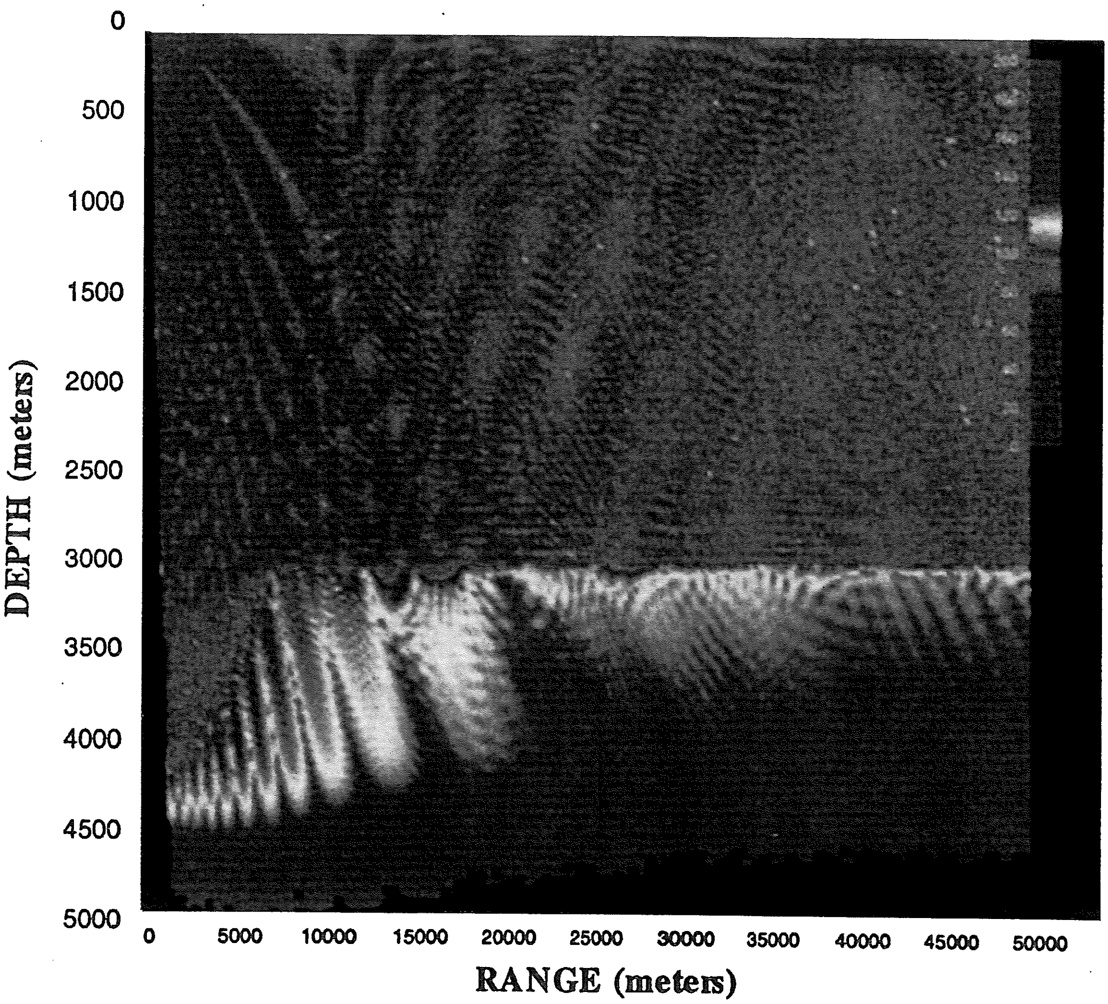

Figure 1. Acoustic intensity (expressed in transmission loss in dB) for the acoustic environoment of Table I, with no loss or scattering. The number of points in the FFT window is 32.

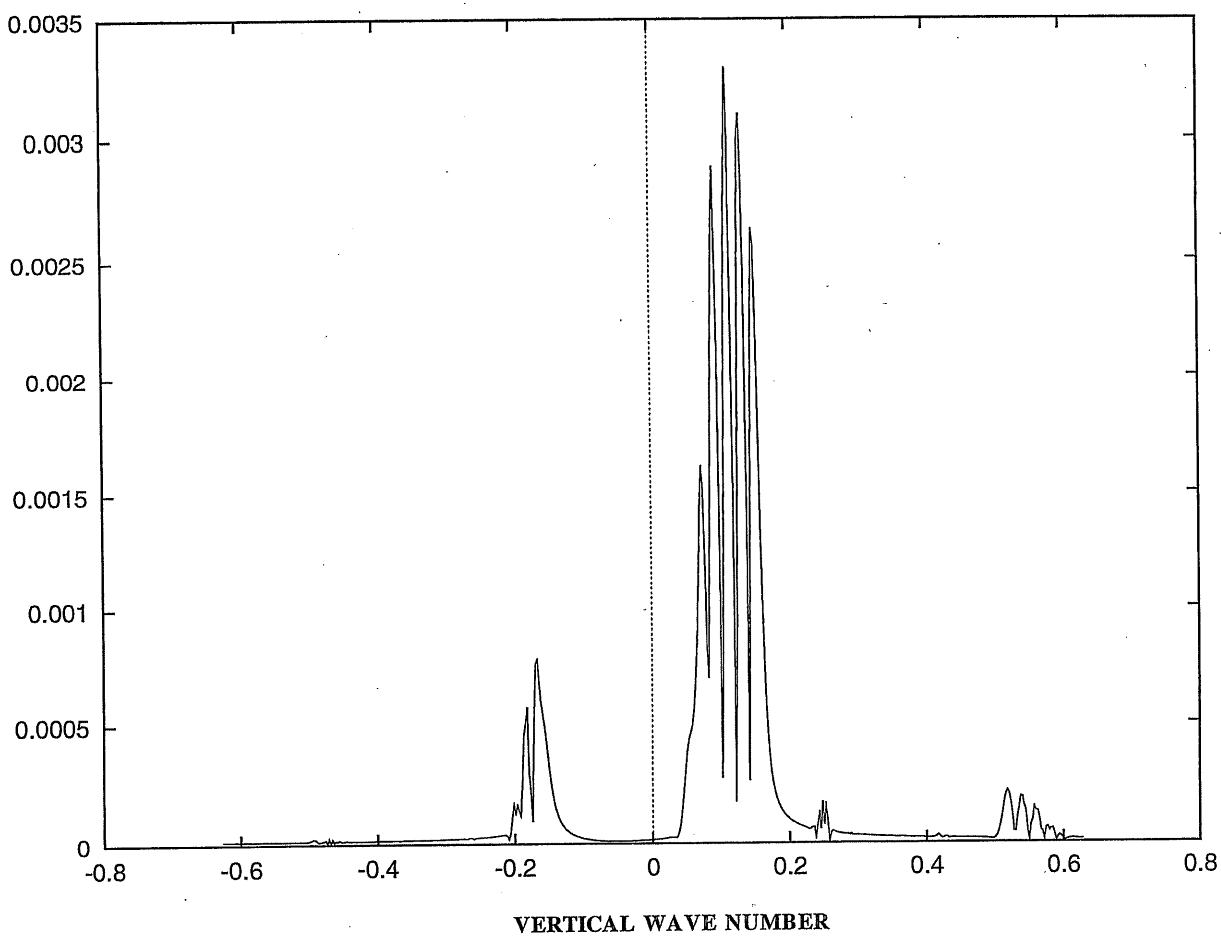

Figure 2. The complex pressure field in k-space in the bottom layer, at a range of 6 Km.

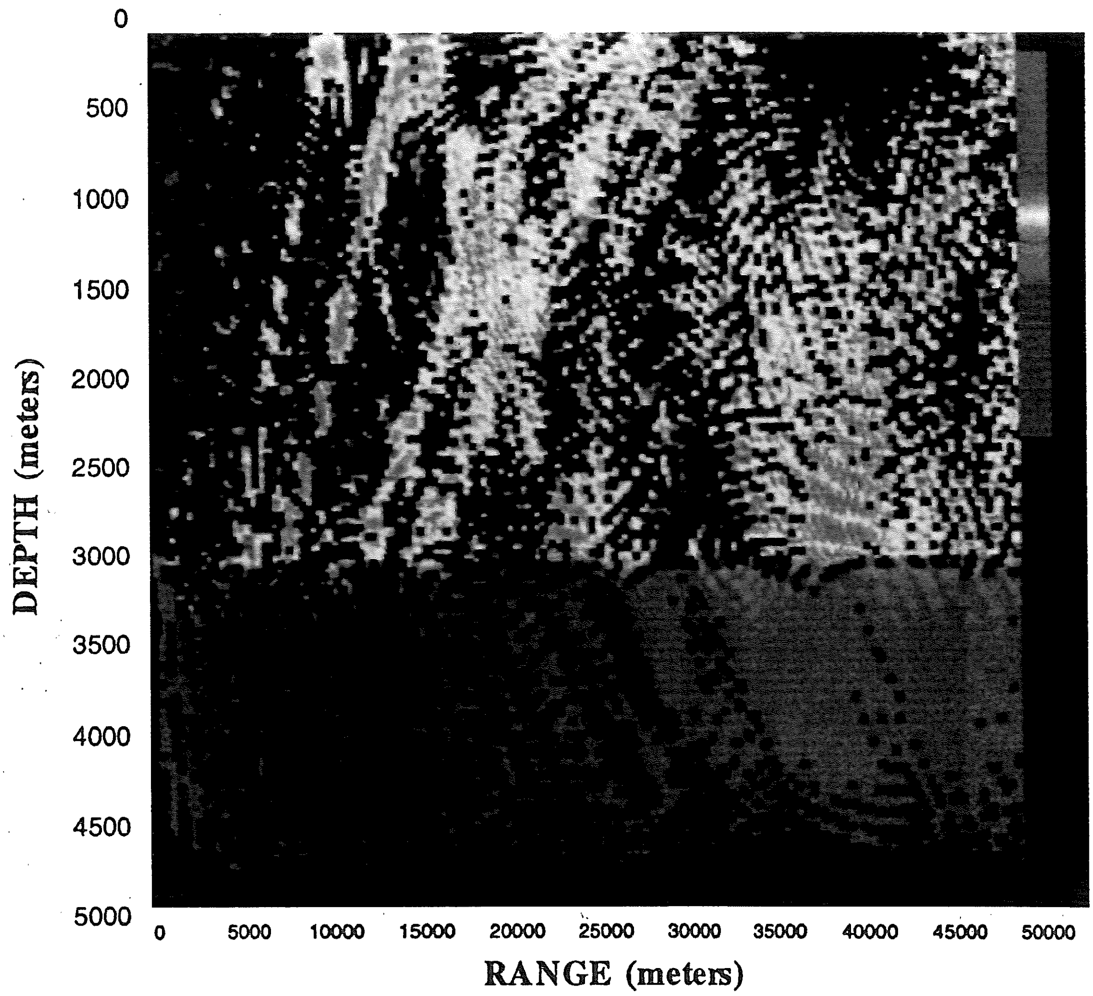

Figure 3. The difference between the acoustic intensities for the case of $\mu = 0$ and $\mu = 0.002$ (-27 dB). Black areas represent a difference in intensities less than or equal to zero.

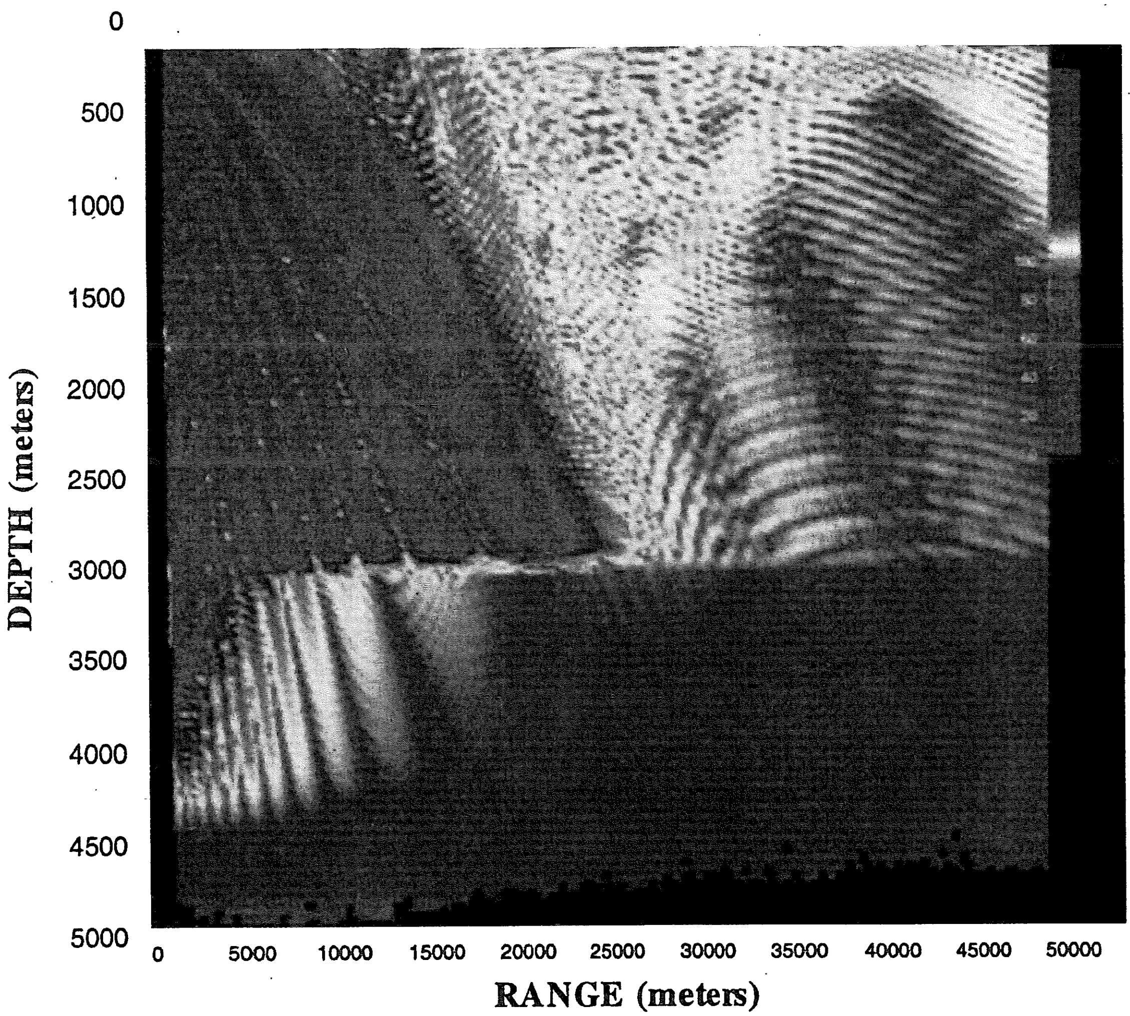

Figure 4. Acoustic intensity with bottom loss implemented. The loss parameter α has the value 10 dB per bottom bounce.

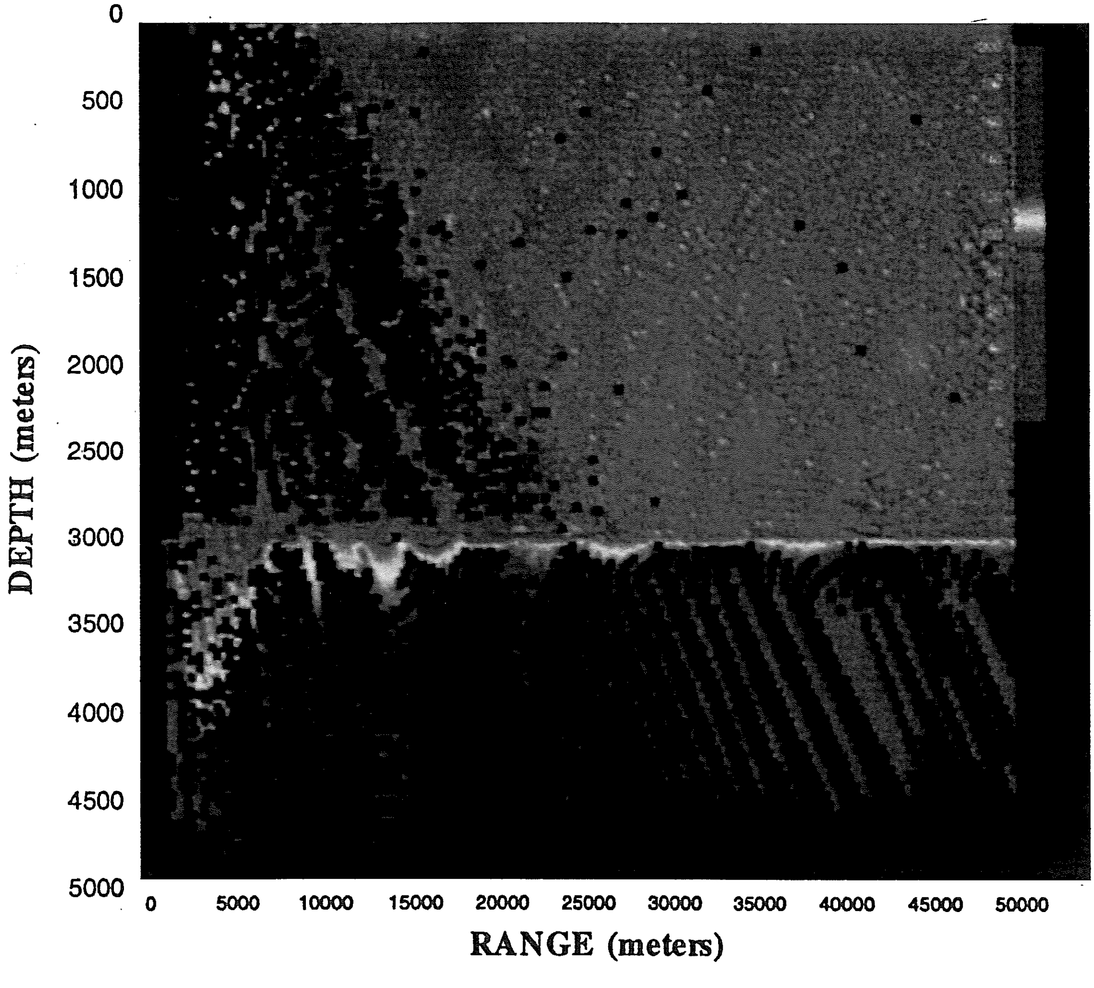

Figure 5. The difference between the acoustic intensities in Figs. 1 and 4.

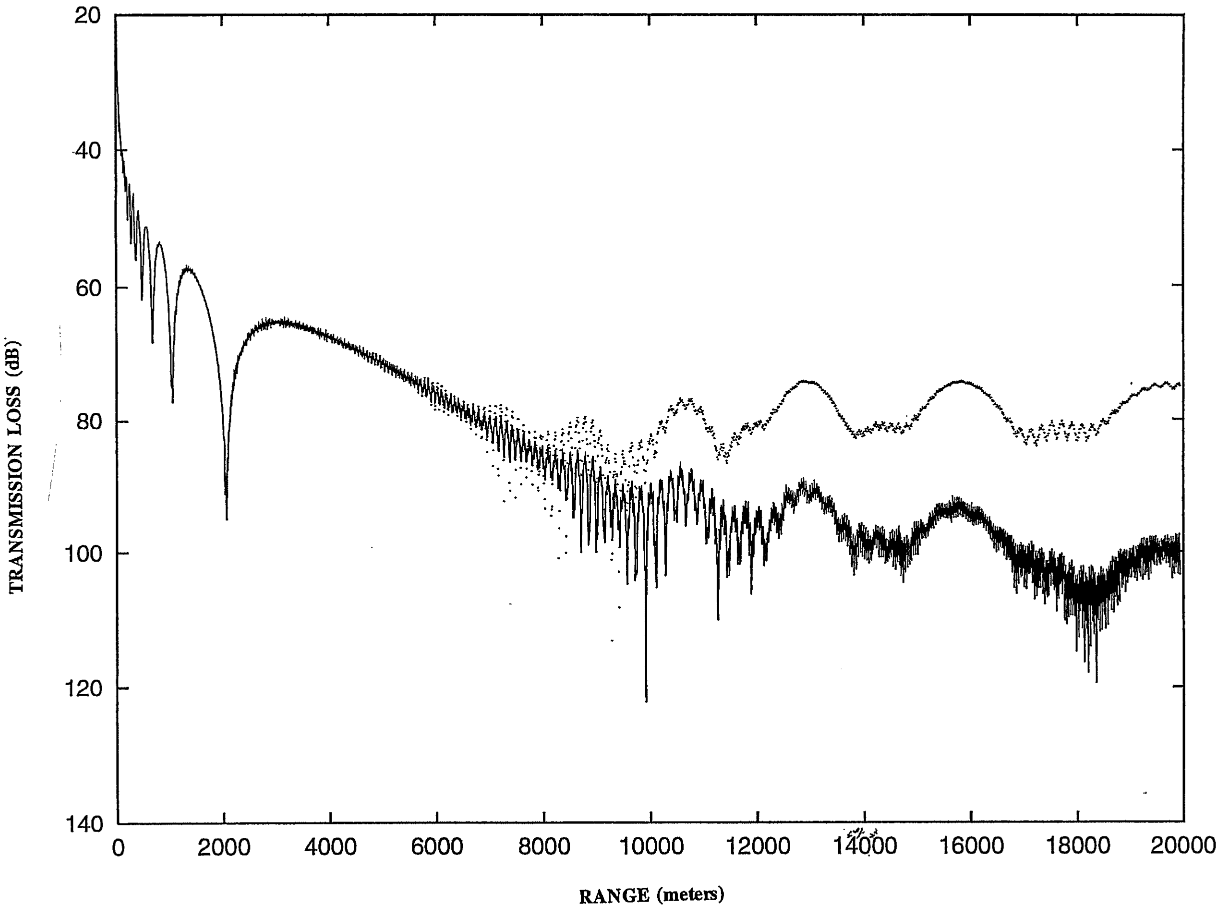

Figure 6. Transmission Loss for a receiver at 150m. The dotted curve represents no loss, the solid curve a loss of 10 dB per bounce.

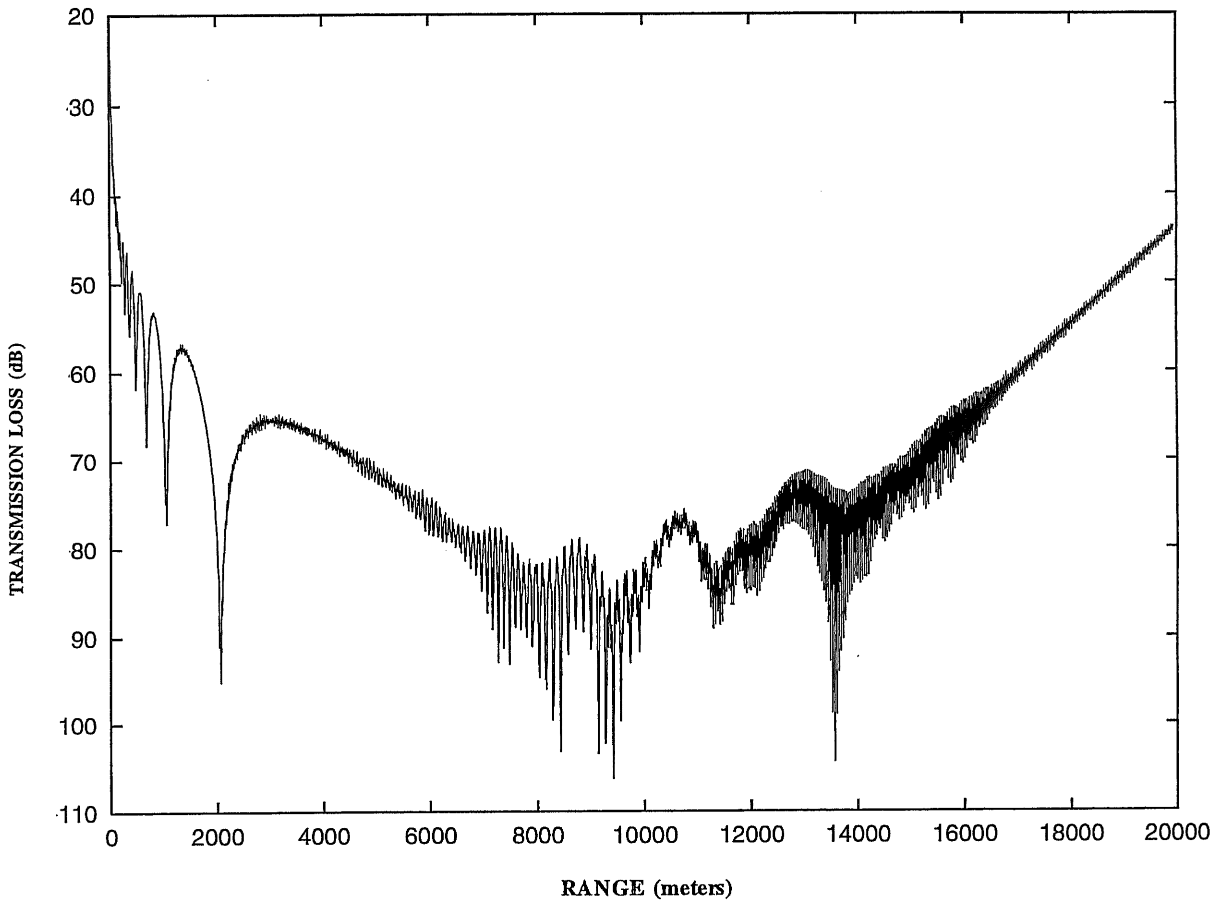

Figure 7. Transmission loss for a receiver at 150m with μ=.005. The result diverges rapidly after 13,000m.

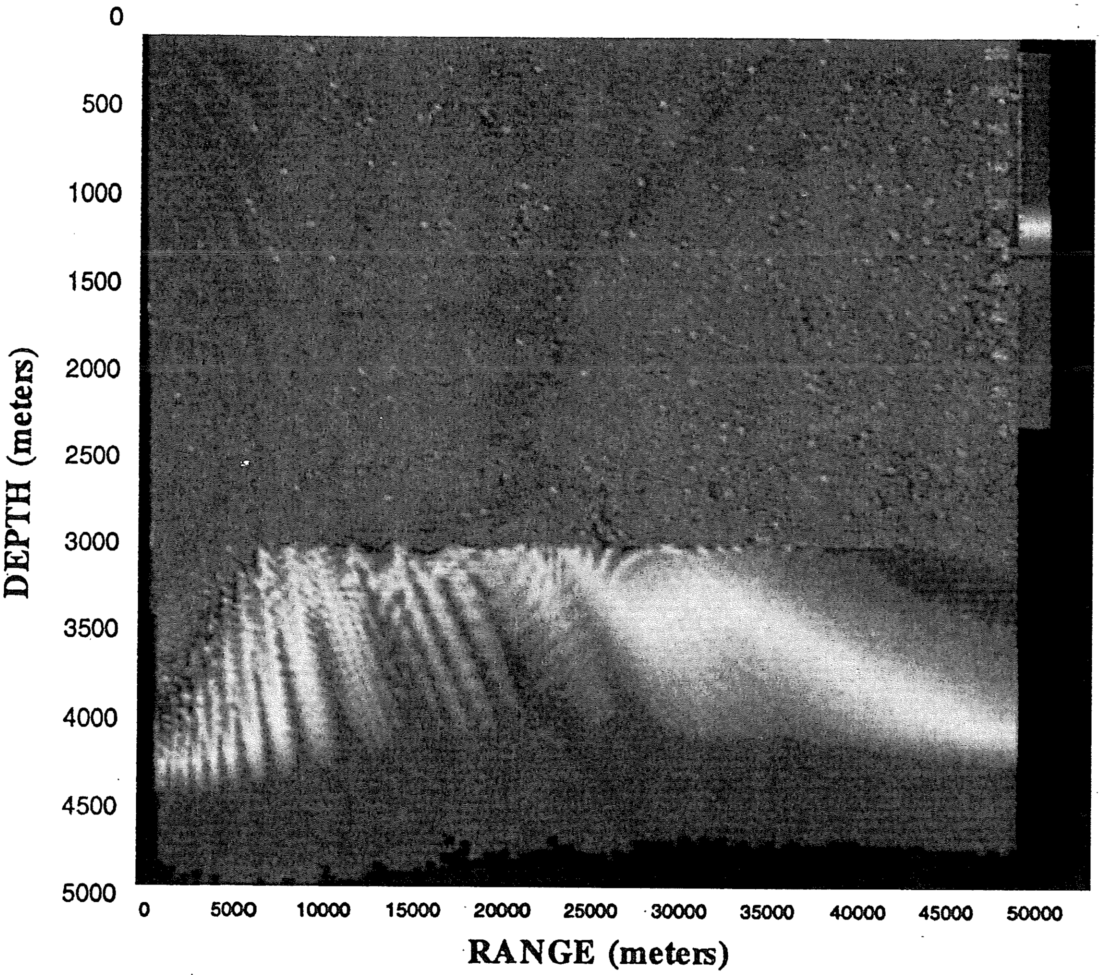

Figure 8. Pressure field for the case of Fig. 7. Note the unphysical values of TL for $R > 10$ km.