

## Neural Network Inversion and Applications to Estimate Some Parameters of a Submerged Moving Body

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### ABSTRACT

This paper presents the concept, principle, and method of neural network inversion. The neural network refers to a particular multi-layered, paralleled data processing system which can adapt the weight matrix to match the changes of the environment. The neural network inversion means to implement the mapping from a multi-dimensional space of observation field to another multi-dimensional parameter's space by this particular system. As one of the applications to the ocean science, the research discusses the detection of some parameters for a moving body that can be seen as a moving acoustic source. A trained neural network has a capability to find the parameters of a submerged moving body with a real time solution.

### 1 INTRODUCTION

The detection of a submerged moving source has attracted attentions for the last decade. Basically, there are two measurement means: acoustics and electromagnetics. As a acoustic approach a source that emits a constant tone and moves at a constant speed can be localized by measurement of the Doppler shifted frequencies (DSF). The acoustic signal is received by spatially separated sensors. There must be five sensors to give a determinant solution. Weinstein (1982) gives an exact solution. Weinstein and Levanon (1980) and Statman and Rodemich (1987) gives iterative solutions, and Chan and Jardines (1990) and Chan and Towers (1992) gives the grid search solution. The high dimensionality leads to a large computational time. Chan (1994) developed a one dimensional grid search solution which requires three sensors and greatly reduces the computational time.

As an electromagnetic approach it is possible to detect the submerged body itself or its wake. Robindon (1992), Marshall (1988) indicated that the sensitivity of electromagnetic field sensors has been improved to allow non-negligible detection ranges. However, the electromagnetic fields produced by the eddy currents in its metal parts which are rotating in the earth magnetic field may not expect to observe more then several hundred meters away from the moving body. Tuck (1994) presented a theoretical study and shown that the motion of sea water due to the wake of a submerged moving body induces significant magnetic signal as far away as ten kilometers along its path. Earlier, it was the general belief that only shot-period waves in a moderately rough sea could

produce a measurable magnetic field. However, Weaver (1965) showed that long-wavelength ocean swell can be as important as local wind waves of great amplitude.

In this paper we present a new approach based on the finite element method (Teng, 1989; Teng, 1993) and neural network inversion (Fei, 1995; Fei, Kuo and Teng, 1995) to localize the submerged moving source. Although geophysicists have successfully applied neural network to detect unknown underground inclusions, designing and training a network are still more of arts than science (Poulton and et al., 1992). Fei (1995) and Fei, Kuo and Teng (1995) present the concept and principle of neural network inversion to detect the spatial parameters of unknown inclusions (Fei, Teng and Kuo, 1994). Moreover, the basic theorem-three-layer neural network existence theorem (Lorentz, 1976; Kolmogorov, 1957; Hecht-Nielsen, 1987; Hecht-Nielsen, 1989; Carrol and Dickinson, 1989; Cybenko, 1989) and multi-layer neural network existence theorem (Fei, Kuo and Teng, 1995) provide a solid theoretical foundation for the neural network inversion.

An acoustic field distribution for a moving source that emits a series pulse with special forms and moves at a constant course and speed can be computed by finite element method. The finite element modeling can simulate unstratified fluids and complex sea bottom structure, this method can generate real data to train the neural nets. The trained network can be used to find a real time solution based the neural network inversion principle. Using the 3-D finite element method to solve problems presented by realistic sea shore structures still requires considerable memory and execution time in computation. In this research, we only provide the numerical data from the 1-D and 2-D finite element modeling.

## 2 FINITE ELEMENT MODELING

We employ finite element method as a training data generator. The wave equation of pressure field  $p$  for a moving source with a constant velocity  $v_0$  is

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} + S\delta(x - x_0 + v_0 t)\delta(z - z_0)$$

$$c^2 = k/\rho$$

where  $\delta$  is Dirac delta function,  $x_0, z_0$  is the source point,  $c$  is the sound speed of acoustic wave,  $k$  is the bulk modulus,  $\rho$  is mass density and  $S$  is the applied source function.

The finite element equation in matrix form is

$$[M]\{\ddot{p}\} + [K]\{p\} = \{f\}$$

where  $[M]$  is the global mass matrix,  $[K]$  is the global pseudo stiffness matrix,  $\{p\}$  is the column matrix for pressure field.  $\{f\}$  is the applied source matrix.

The time integration-explicit central difference scheme is

$$\{p(t + \Delta t)\} = \{\dot{p}(t) + \{p(t)\} \Delta t$$

$$\{\dot{p}(t + \Delta t)\} = \{\dot{p}(t) - [K][M]^{-1}\{p(t + \Delta t)\} \Delta t$$

The Figure 1 indicates the source function for this moving source. While in the conversational methods the moving source emits a constant frequent tone. The two kinds of sources can be converted by Fourier transformation.

## 3 NEURAL NETWORK AND NEURAL NETWORK INVERSION

The term neural network (or artificial neural network), is derived from its resemblance to the biological interconnection of neurons. A typical neural network consists of many processing elements,

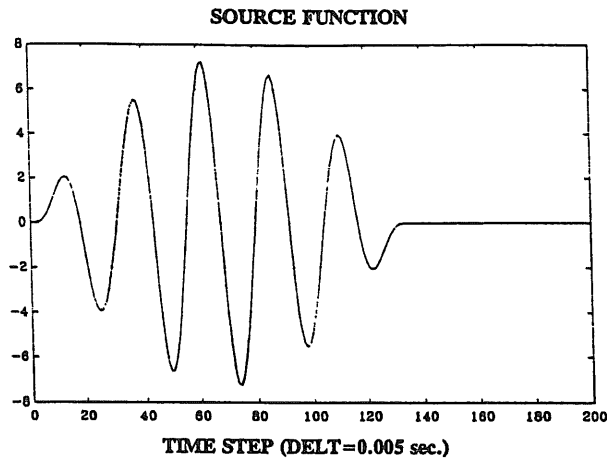


Figure 1: The source function of a moving source.

each of which is tightly connected to another by the modifiable weights. Neural network is expected to learn by representative sets of examples from the given condition.

The neural network may be defined as:

**Definition** The neural network is defined as an overly simplified human-brain system with a parallel, distributed information processing structure, which consists of a number of layered processing elements. Each element in a given layer has a single output connection into all elements in the next layer. When the information is sensed by analogy to, say, the eyes, ears or skin, it is passed on to the multi-layer nerve system. Comparison with the prior information as the trained network keeps the learning results as a fixed weigh matrix, the neural network thus discriminates the output of the results, just as if the human-brain system has the capability of judgment.

In this paper we simply regard "neural network" as a parallel, three-layer data-processing machine without real biological meaning attached. A three-layer neural network configuration consists of one input, one hidden, and one output layer. Every element (or neuron) is connected to the next layer by the so called "weights." The weights are calculated during a supervised training process, in which a representative set of input and a desired output is presented to the network. The network undergoes a learning procedure to register all changes of weights through the feed-forward and backpropagation neural networks cyclically. One cycle (or one iteration) is to complete a feed-forward and backpropagation training procedure.

For a given sea water model with a moving source, the finite-element modeling method can simulate the distribution of the wave-field on the sea water as

$$p(\mathbf{m}, x_r, t, n)$$

where  $\mathbf{m}$  denotes the source position,  $x_r$  denotes the recording point,  $t$  is the time, and  $n$  denotes environment parameters of the sea.

The meaning of inversion here is to seek a set of parameters  $\mathbf{m}$  that minimizes some norm of the difference between the observed data  $d$  and the predicted data  $p$ . In general, the  $L_2$  norm of the data mismatch is minimized, leading to the following objective function,

$$F(\mathbf{m}) = |d - p(\mathbf{m})|^2 = |e|^2.$$

The solution for an inverse problem is defined as a set of parameters  $\mathbf{m}$  for which  $F(\mathbf{m})$  is the global minimum of the function  $F$ .

The objective function is generally minimized interactively by updating the subsequent initial guess of the position parameter  $\mathbf{m}_0$ .

Assume the unknown true value is  $\mathbf{m}_t$ . Set up an  $n$ -dimensional Euclidean space with coordinates  $M_1, M_2, \dots, M_n$ . The initial guess  $\mathbf{m}_0$  and the unknown true value  $\mathbf{m}_t$  are mapped into the  $n$ -dimensional space. The inversion procedure can be seen as a point movement from  $\mathbf{m}_0$  toward  $\mathbf{m}_t$ . We may use the finite-element method to forward-model  $\mathbf{p}(\mathbf{m}_0)$ , and calculate the error function such that

$$|\mathbf{d} - \mathbf{p}(\mathbf{m}_0)|^2 = |\mathbf{e}|^2$$

Therefore, we adopt an interactive inversion method to move the point  $\mathbf{m}_0$  through  $m_1 \dots m_n$ . The distance and direction of the movement are related to the adopted method. By means of minimization, the error function approaches a minimum  $e_0 \geq e_1 \geq e_2 \dots \geq e_n \rightarrow 0$ . When  $|\mathbf{e}|$  is smaller than a given positive value, this interactive procedure terminates. This global minimum is empirically assured in a conventional inversion method, when the initial point  $\mathbf{m}_0$  is in the valley of the global minimum, the search will successfully reach the point  $\mathbf{m}_t$ . When the initial point  $\mathbf{m}_0$  is trapped in the valley of the local minima, the search fails.

We choose  $k$  sampling points  $\mathbf{m}_1, \mathbf{m}_2, \dots, \mathbf{m}_k$ , which are uniformly distributed in the sphere. Then we establish a matrix  $\mathbf{p}_{tm}$  to train the neural network, where

$$\mathbf{p}_{tm} = \mathbf{p}(\mathbf{m}, x_r, t).$$

The training procedure can be viewed as the confirmation of the mapping between  $\mathbf{p}_{tm}$  and  $\mathbf{m}$ . After learning, the neural network assumes the capability to inverse the real observed data  $\mathbf{p}_{tm}$  to find the  $\mathbf{m}_t$ .

## 4 NEURAL NETWORK TRAINING

In the feed-forward neural network, The input data points of the vector  $x_{i_1}^{(1)}$  is first presented to the  $l_1$  input layer of the neural units so that the after-sigmoidal output at the  $l_2$  later (or hidden layer) is

$$y_{i_2}^{(2)}(\mathbf{x}) = \sigma\left(\sum_{i_1=1}^{I_1} W_{i_1 i_2}^{(1,2)} x_{i_1}^{(1)}\right) = \sigma(S_{i_2}^{(2)})$$

which, in turn, is used as the input to the  $l_2$  layer.

The after-sigmoidal output at the  $l_3$  output layer is

$$y_{i_3}^{(3)}(\mathbf{x}) = \sigma(S_{i_3}^{(3)}(\mathbf{x})) = \sigma\left(\sum_{i_2=1}^{I_2} W_{i_2 i_3}^{(2,3)} \sigma\left(\sum_{i_1=1}^{I_1} W_{i_1 i_2}^{(1,2)} x_{i_1}^{(1)}\right)\right).$$

In above two equations,  $W_{i_1 i_2}^{(1,2)}$  and  $W_{i_2 i_3}^{(2,3)}$  are the weights, linking the  $l_1$  and  $l_2$  layer, and the  $l_2$  and  $l_3$  layer, respectively,  $\sigma$  is a sigmoidal function,  $i_1=1, 2, \dots, I_1$ , and  $i_2 = 1, 2, \dots, I_2$  are the neuron's numbers in the  $l_1$  and  $l_2$  layer, respectively.

In the feed-forward neural network,  $x_{i_1}^{(1)}$  is known, and in the initial feed-forward (or the first iteration),  $W_{i_1 i_2}^{(1,2)}$  and  $W_{i_2 i_3}^{(2,3)}$  are made to be random, and  $\sigma$  is the sigmoidal function.

$$\sigma(s) = \frac{1}{1 + e^{-s}}.$$

The backpropagation is essentially a generalization of the least-squares procedure for the neural network between the input and output layers. The change of the weight is calculated starting on

the output layer and going backward calculating updated weights for each layer. In the process, the so-call "delta-rule" is applied to increase the speed of convergence to stable state.

In training, the post-sigmoidal error  $e_{i_3}^{(3)}[t]$  at the  $l_3$  layer is then backpropagated to the  $l_3$  layer,

$$e_{i_3}^{(3)}[t] \leftarrow y_{i_3}^{(3)'} - y_{i_3}^{(3)}$$

where  $y_{i_3}^{(3)'}$  is the known desired output.

The post-sigmoidal error in the  $l_2$  (or hidden) layer through backpropagation is then:

$$e_{i_2}^{(2)}[t] \leftarrow \sum_{i_3=1}^{I_3} W_{i_2 i_3}^{(2,3)}[t] \delta_{i_3}^{(3)}[t],$$

where  $\delta_{i_3}^{(3)}[t]$  is obtained by:

$$\delta_{i_3}^{(3)}[t] \leftarrow \frac{\partial \sigma(p_{i_3}^{(3)}[t])}{p_{i_3}^{(3)}[t]} e_{i_3}^{(3)}[t],$$

and

$$p_{i_3}^{(3)}[t] \leftarrow \sum_{i_2=1}^{I_2} W_{i_2 i_3}^{(2,3)}[t] q_{i_2}^{(2)}[t].$$

The after-sigmoidal input  $q_{i_2}^{(2)}[t]$  at the  $l_2$  layer is simply

$$q_{i_2}^{(2)}[t] = \sigma(S_{i_2}^{(2)}) = \frac{1}{1 + e^{-S_{i_2}^{(2)}}},$$

and,  $W_{i_2 i_3}^{(2,3)}$  is obtained from the initial feed-forward in being random.

Again, here  $W_{i_2 i_3}^{(2,3)}[t]$  is the random weights assigned in the initial feed-forward network.

At this stage the change of the weights between neuron  $i_3$  in the  $l_3$  layer and every neuron  $i_2$  in the  $l_2$  layer can be calculated by:

$$\Delta W_{i_2 i_3}^{(2,3)}[t] \leftarrow \beta \delta_{i_3}^{(3)}[t] q_{i_2}^{(2)}[t]$$

and the modified weight with the addition of the momentum term

$$W_{i_2 i_3}^{(2,3)}[t+1] = W_{i_2 i_3}^{(2,3)}[t] + \beta \delta_{i_3}^{(3)}[t] q_{i_2}^{(2)}[t] + \alpha \Delta W_{i_2 i_3}^{(2,3)}[t-1],$$

where  $\beta$  is the learning rate and  $\alpha$  is the momentum rate.

Following the same procedure, we are able to backpropagate to obtain the post-sigmoidal error  $e_{i_1}^{(1)}[t]$ , the change of the weight  $\Delta W_{i_1 i_2}^{(1,2)}[t-1]$  and the modified weight  $W_{i_1 i_2}^{(1,2)}[t+1]$  linking the  $l_2$  to  $l_1$  layer.

For the second iteration (or the second cycle), the second feed-forward output at the  $l_3$  layer  $y_{i_3}^{(3)}(\mathbf{x})$  can be obtained by the feed-forward process, using the weights  $W_{i_1 i_2}^{(1,2)}$  and  $W_{i_2 i_3}^{(2,3)}$  having been updated in the first iteration, and the input data points of the vector  $x_{i_1}^{(1)}$ . By the same token the post-sigmoidal error  $e_{i_3}^{(3)}$  at the  $l_3$  layer is backpropagated through the  $l_3$ ,  $l_2$ , and  $l_1$  layer to complete the second iteration (or the second cycle), and to obtain the newly updated modified weights  $W_{i_1 i_2}^{(1,2)}$  and  $W_{i_2 i_3}^{(2,3)}$ .

Such an iterative feed-forward and backpropagation procedure is repeatedly carried out to obtain the final optimized weights.

If the network is properly trained with a representative set of acoustic data, the final inversion is accomplished by the feed-forward network to obtain the after-sigmoidal output at the  $l_3$  layer that is the most optimized output in the sense of least-square-error.

Table 1. The velocity inversion for 1-D modeling.

No	NNT inversion	True value	Accuracy	NNT inversion	True value	Accuracy
	unit(km)	unit(km)	%		unit(km)	%
1	20.23	20	1.13	19.76	20	-1.2
2	25.03	25	0.11	25.05	25	0.22
3	29.92	30	-0.25	30.16	30	0.53
4	35.04	35	0.11	35.03	35	0.07
5	39.98	40	-0.05	39.97	40	-0.06
6	44.92	45	-0.18	45.02	45	0.04
7	49.94	50	-0.12	49.92	50	-0.16
8	54.99	55	-0.01	54.99	55	-0.02
9	60.03	60	0.05	59.94	60	-0.1
Average			0.22			0.26
Position=12.5km				Position=13km		

No	NNT inversion	True value	Accuracy	NNT inversion	True value	Accuracy
	unit(km)	unit(km)	%		unit(km)	%
1	20.06	20	0.31	19.81	20	-0.97
2	25.1	25	0.41	25.09	25	0.37
3	29.82	30	-0.59	29.98	30	-0.08
4	34.95	35	-0.13	35.04	35	0.11
5	39.99	40	-0.02	39.99	40	-0.03
6	45.05	45	0.12	44.98	45	-0.04
7	49.84	50	-0.32	50	50	0.01
8	55.03	55	0.06	55.08	55	0.15
9	59.99	60	-0.01	60.06	60	0.1
Average			0.21			0.21
Position=13.5km				Position=14km		

No	NNT inversion	True value	Accuracy	NNT inversion	True value	Accuracy
	unit(km)	unit(km)	%		unit(km)	%
1	20.24	20	1.18	20.11	20	0.54
2	24.94	25	-0.22	25.05	25	0.2
3	29.87	30	-0.43	29.89	30	-0.36
4	35	35	-0.01	35.05	35	0.13
5	39.96	40	-0.09	39.87	40	-0.32
6	45.04	45	0.09	45.05	45	0.1
7	49.96	50	-0.08	50	50	0
8	55.11	55	0.2	54.95	55	-0.08
9	60.06	60	0.09	59.98	60	-0.03
Average			0.26			0.2
Position=14.5km				Position=15km		

## 5 APPLICATIONS

Figure 2 shows the 1-D finite element modeling of a moving source. The acoustic velocity of the sea water is 1.5 km/s. The x-coordinate of the receiver is 0 km. The x-coordinate of the start point is from 12 to 18 km at an interval 0.5 km. The velocity of the moving source is 5, 10, ... 60 km/h.

Figure 3 illustrates the training data. The training model number 1 to 12 denotes the velocities of the moving source from 5 to 60 km/h at an interval 5 km/h. Figure 3(a). The start point position of the moving source is 12 km. Figure 3(b). The start point position of the moving source is 14.5 km. Figure 3(c). The start point position of the moving source is 17 km.

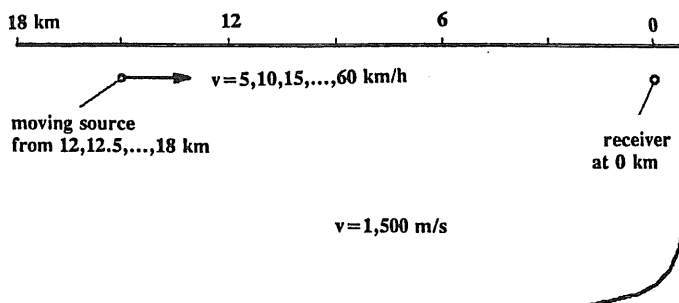


Figure 2: 1-D finite-element model. The acoustic velocity of the sea water is 1.5 km/s. The x-coordinate of the receiver is 0 km. The x-coordinate of the start point is from 12 to 18 km at an interval 0.5 km. The velocity of the moving source is 5, 10, ... 60 km/h.

Table 1 indicates the result of neural network inversion for the 1-D moving source for the position of 12.5, 13, 13.5, 14, 14.5, 15 km. The average accuracy is 0.22%.

Figure 4 is 2-D finite-element modeling for a moving source. The acoustic velocity of sea water is 1.5 km/s. The thickness of the layer is 1000m. The second layer's velocity is 2.7 km/s. The depth of the moving source is 200 m, and the start position is 0 km. The x-coordinate of the receivers on the surface of the sea-air is 0, 0.5, ... 6.0 km. The velocity of the moving source is from 5 km/h to 60 km/h at an interval 5 km.

Figure 5 shows the finite element synthetic seismogram. Figure 5(a). The velocity of the moving source is 20 km/h. Figure 5(b). The velocity of the moving source is 40 km/h. Figure 5(c). The velocity of the moving source is 60 km/h.

Figure 6 shows the training data, which is collected from the results of the finite element method. The training model number 1 to 12 denotes the velocities of the moving source from 5 to 60 km/h at an interval 5 km/h. Figure 6(a). The velocity of the moving source is 5 km/h. Figure 6(b). The velocity of the moving source is 30 km/h. Figure 6(c). The velocity of the moving source is 55 km/h.

Table 2 shows the detection of velocity for 2-D finite element modeling. For the position of 5, 10, 15, 25 km, the average accuracy of the neural network inversion is 2.7%, 3.7%, 2.2%, 2.36%, respectively.

## 6 CONCLUSIONS

1) A combination of the finite-element forward modeling method and the neural network inversion method proves to be an efficient tool to deal with the detection of speed and position of a moving source in the sea water. The sea water can be unstratified fluids and the host medium can be a complex structure. The finite-element forward modeling is essential to provide accurate, reliable and representative sets of data in the neural network training.

2) Although this paper deals with the moving source in 1-D and 2-D sea water environment, the treatment is equally applicable to 3-D resistively problems.

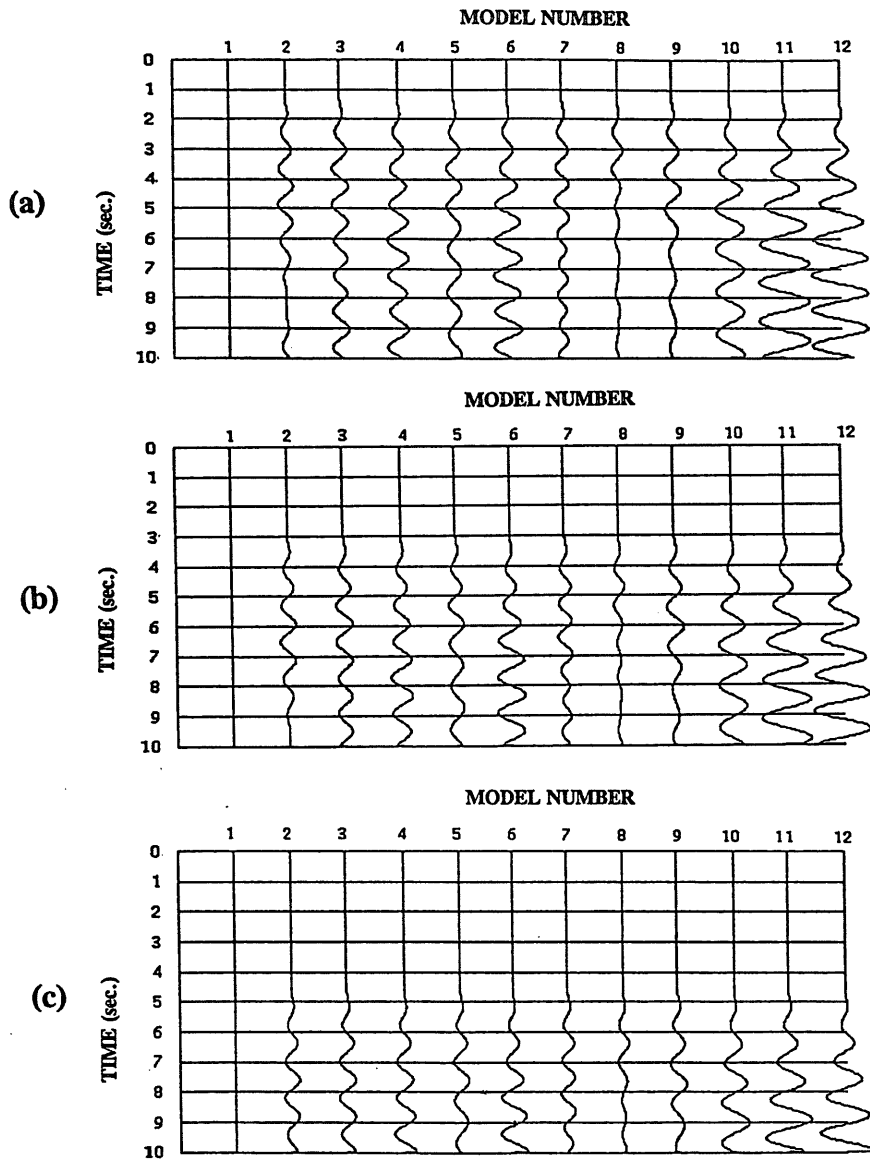


Figure 3: The training data. The training model number 1 to 12 denotes the velocities of the moving source from 5 to 60 km/h at an interval 5 km/h. a) The start point position of the moving source is 12 km. b) The start point position of the moving source is 14.5 km. c) The start point position of the moving source is 17 km.



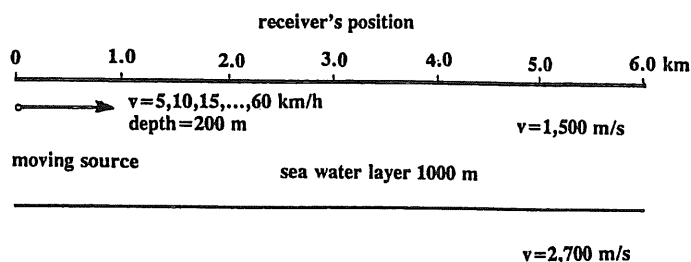


Figure 4: 2-D finite-element model. The acoustic velocity of the sea water is 1.5 km/s. The thickness of the first layer is 1000 m. The second layer's velocity is 2.7 km/s. The depth of the moving source is 200 m, and the start position is 0 km. The x-coordinate of the receivers on the surface of the sea-air is 0, 0.5, ..., 6.0 km. The velocity of the moving source is from 5 km/h to 60 km/h at an interval 5 km.

Table 2. The velocity inversion for 2-D modeling.

No	NNT inversion unit(km)	True value unit(km)	Accuracy %	NNT inversion unit(km)	True value unit(km)	Accuracy %
1	21.75	20	8.75	17.13	20	-14.36
2	24.1	25	-3.6	24.19	25	-3.22
3	30.88	30	2.95	28.69	30	-4.36
4	35.17	35	0.48	34.38	35	-1.76
5	39.19	40	-2.03	42.26	40	5.66
6	45.44	45	0.97	45.35	45	0.77
7	48.68	50	-2.65	49.56	50	-0.88
8	56.33	55	2.42	54.21	55	-1.43
9	59.72	60	-0.47	60.96	60	1.61
Average			2.7			3.7

Position=5km

Position=10km

No	NNT inversion unit(km)	True value unit(km)	Accuracy %	NNT inversion unit(km)	True value unit(km)	Accuracy %
1	21.17	20	5.85	14.22	20	***
2	24.52	25	-1.92	24.49	25	-2.06
3	30.03	30	0.1	29.59	30	-1.36
4	33.85	35	-3.29	34.91	35	-0.25
5	39.43	40	-1.44	40.07	40	0.19
6	46.92	45	4.26	46.02	45	2.26
7	50.84	50	1.69	53.16	50	6.32
8	54.39	55	-1.11	52.11	55	-5.25
9	60.06	60	0.1	60.72	60	1.21
Average			2.19			2.36

Position=15km

Position=25km

## 7 ACKNOWLEDGEMENTS

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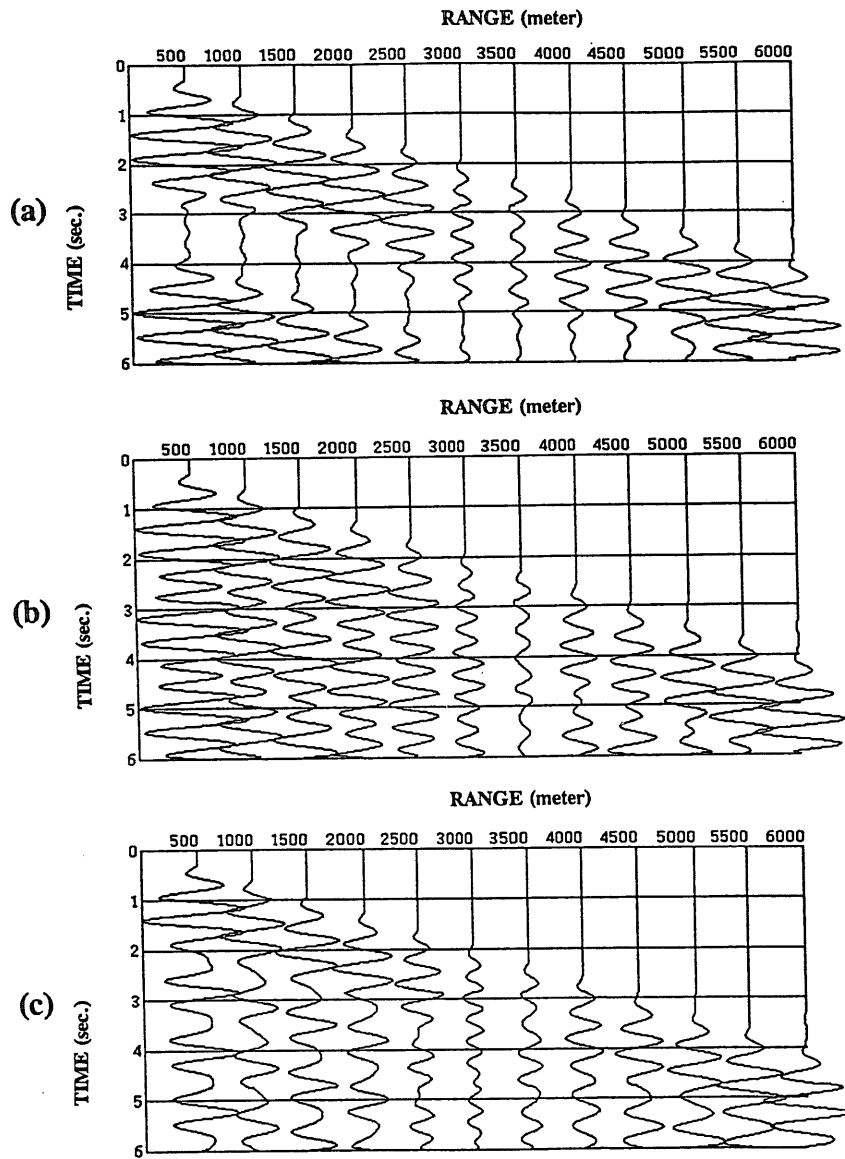


Figure 5: The finite element synthetic seismogram. a) The velocity of the moving source is 20 km/h. b) The velocity of the moving source is 40 km/h. c) The velocity of the moving source is 60 km/h.

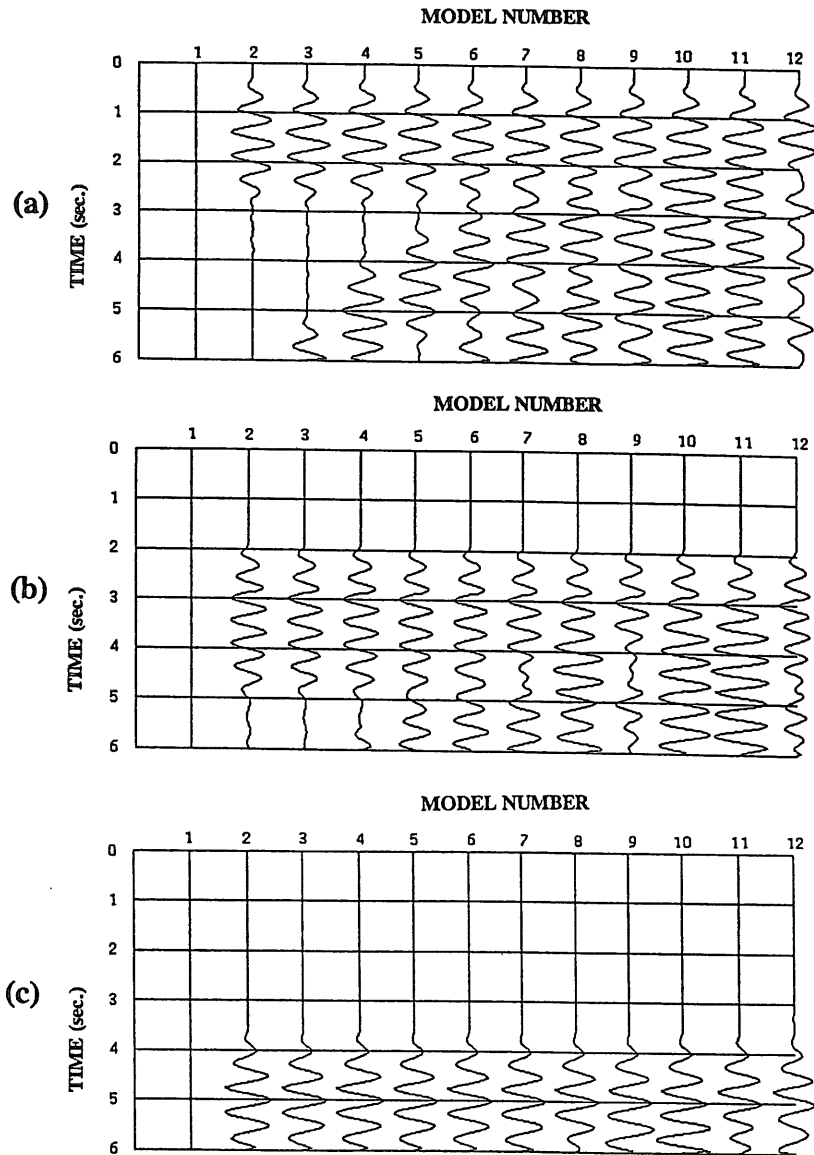


Figure 6: The training data collected from the computation results of the finite element method. The training model number 1 to 12 denotes the velocities of the moving source from 5 to 60 km/h at an interval 5 km/h. a) The velocity of the moving source is 5 km/h. b) The velocity of the moving source is 30 km/h. c) The velocity of the moving source is 55 km/h.

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