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Three Neural Inverse Methods for Ocean Acoustic Tomography

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Abstract

Ocean Acoustic Tomography (OAT) is an oceanographic method for the global observation of the ocean. All the concepts rely on the ability of the ocean as a multipath sound guiding system on long ranges. Acoustical data are used to map the structure of the sound velocity field in both time and space. Usually, linear inversions are performed. They use a linearization around a reference environment i.e. the real sound velocity field is considered to be the sum of the reference field and a first order perturbation field. The difference between travel times of rays computed in the reference environment and the measured travel times (travel time anomaly) of the same rays is inverted to retrieve the sound speed perturbations. We present in this paper three non linear inversion methods based upon the use of neural networks: direct inversion, network (or adjoint) inversion and distal inversion. The methods are applied in the case of simulated data in a Mediterranean environment. They are compared with a usual linear method (based upon perturbations around a reference environment).

1 INTRODUCTION

Ocean Acoustic Tomography (OAT) is an oceanographic method for the global observation of the ocean [1]. All the concepts rely on the ability of the ocean as a multipath sound guiding system on long ranges. Acoustical data are used to map the structure of the sound velocity field in both time and space. More precisely, the estimation of the sound-velocity is obtained by inverting the arrival times of a given signal emitted from a fixed source to a fixed receiver at a fixed range. Usually, linear inversions are performed. They use a linearization around a reference environment, i.e. the real sound velocity field is considered to be the sum of the reference field and a first order perturbation field. The difference between travel times of rays computed in the reference environment and the measured travel times (travel time anomaly) of the same rays is inverted to retrieve the sound speed perturbations. For these inversions, the construction of a model mapping the sound speed perturbation onto the travel time anomaly is required and two major hypothesis are assumed. First, good a prior information about the reference environment has to be known to track and identify arrivals. Second, it is assumed that the paths of the rays in the reference environment are not altered by the perturbation of the sound speed environment. These hypothesis can yield to errors and biases in inversion results.

A new approach, avoiding the latter assumption has been proposed recently using neural networks [2]. It relies on the ability of neural networks to approximate non-linear functions. Given a model for the forward problem, a neural network is used to approximate the inverse solution by simply reversing the role of input and output parameters (direct inversion). We propose in this paper alternative solutions to perform indirect inversion with neural nets. The first approach is referred to the adjoint inversion method). It consists of approximating the forward function with a multilayer Perceptron, to freeze weights and to apply a backpropagation-to-input algorithm (instead of backpropagation-to-weights as in the direct method). This scheme can be related to adjoint-based inversion methods used in Meteorology and Oceanography. The second approach is a combination of the two previous ones. It consists in composing both the inverse and forward problem to retrieve the identity function. It is referred to the distal inversion.

The paper is organized as follows: Section 2 gives some background on acoustic tomography and describes the inverse problem to be solved. Section 3 describes the three inverse methods. Section 4 presents the results in the case of simulated data in single slice tomography. The results are compared to those obtained by a usual linear scheme.

2 THE INVERSE PROBLEM IN SINGLE-SLICE ACOUSTIC TOMOGRAPHY

Neglecting the oceanic current effect, the travel time t_i of the acoustic signal along the path R_i is given by:

$$t_i = \int_{R_i} \frac{ds}{c(r,z)} \quad (1)$$

where r and z denotes respectively the horizontal and vertical axis, $c(r,z)$ is the sound velocity field between the source and the receiver and s is the curvilinear abscissa. The relationship between an arrival time and the sound speed field is denoted G_i. It is non-linear and takes the form of:

$$G_i(c(r,z)) = \int_{R_i} \frac{ds}{c(r,z)} \quad (2)$$

Denoting by N the number of rays, the problem can be written in term of a set $G = (G_1, G_2, ..., G_N)$ of non-linear equations so that:

$$t_i = G_i(c(r, z)) \quad (3)$$

The general inverse problem in OAT consists in retrieving an estimation of the sound-speed environment from the time measurements $\mathbf{t}=(t_i)_{i=1,..,N}$. The usual inversion methods are based upon a linearization around a reference environment :The real sound velocity field is considered to be the sum of the reference field and a first order perturbation field. The difference between travel times of rays computed in the reference environment and the measured travel times (travel time anomaly) of the same rays is inverted to retrieve the sound speed perturbations [3, 4, 5].

3 THREE NEURAL SCHEMES FOR INVERSION

To avoid the linearity hypothesis, we propose in this paper "by learning" inversion methods. They rely on the ability of neural networks to approximate non-linear functions.

3.1 Multilayer Perceptron (MLP) for multidimensional approximation

Connectionist models rely upon the behavior of a set of interconnected elementary units (neurons). The global properties of so-called Artificial Neural Networks depend on the internal structure of the model. The main properties used in function approximation are learning and retrieving ability: a neural net is able to learn a relationship from a given set of input-output patterns and to generalize (i.e approximate) the global relationship for unlearned input patterns. Multilayer Perceptrons are a particular class of neural networks in which neurons are connected in layers. One layer is receiving the system input (input layer). One layer is providing the network response to the input (output layer). One or more layers are internal to the network (hidden layers). The layers are fully connected (each neuron of a layer is connected to every neuron of the next layer) and the signals propagate from input to output (no backward information). Biases (additional weights connected to a constant input) can be added on the output and hidden neurons in order to reinforce the non-linear behavior of the network. In this work, the transfer function of hidden units is sigmoidal and the transfer function of the output units is linear. The problem of multidimensional approximation can be posed as follows [6]: Given a real function F:
$F : \mathcal{R}^Q \rightarrow \mathcal{R}^P$
$\mathbf{x} = (x_1, .., x_Q) \rightarrow \mathbf{y} = (y_1, .., y_P) = F(\mathbf{x})$,
and a set of example behaviors of F (i.e a set $\mathcal{L}$ of N examples (x^0, y^0)), the problem is to find a function $G(W, \mathbf{x})$ which gives the best approximation of F on $\mathcal{L}$. W is a vector of internal parameters of the approximating function G. This can be achieved by finding a configuration W^* such as:

$$||G(W^*, \mathbf{x}) - F(\mathbf{x})|| \leq ||G(W, \mathbf{x}) - F(\mathbf{x})|| \quad (4)$$

for all W. The notation $||...||$ stands for an arbitrary norm in $\mathcal{R}^Q$. This definition is general and is not specially related to the neural net concept. However, it can be easily interpreted in the neural framework: the approximation function $G(W, \mathbf{x})$ generalizes the function F from a set of learned examples $\mathcal{L}$ ($\mathcal{L}$ is called the learning base). For this approximation task, the MLP is built as follows. Each x-component is encoded on one neuron of the input layer. Each y-component is encoded on one neuron of the output layer. A cost function, measuring the sum-squared error between the output of the network and the values of F is defined as follows:

$$E(W) = \frac{1}{2} \sum_{m=1}^{N} \sum_{j=1}^{P} [y_{j,m}(W, x_m^0) - y_{j,m}^0]^2 \tag{5}$$

The optimal weight configuration is found by minimizing the cost function using a stochastic backpropagation algorithm [7]. Theoretical works have been done to demonstrate the ability of MLP to approximate continuous multidimensional function defined on compact subsets [8, 9, 10]. Given a function F, there exists a multilayer neural network with one or more hidden layers able to approximate F in the uniform convergence sense. Nevertheless, there are no absolute rules to determine the optimal architecture of the network (number of hidden layers, number of cells in each layer). It must be done heuristically. Approximating a function by increasing the number of hidden units is always possible. However, to be statistically relevant, the approximation has to be done on a number of examples "much greater" than the number of weights. Generally, arbitrarily increasing the number of hidden unit (i.e. the number of internal parameters in the net) generally leads to poor performances in generalization.

3.2 Multilayer Perceptrons for tomographic inversions

3.2.1 Direct inverse modeling

The general scheme of Direct Inverse Modeling (DIM) is given in figure 1. This approach simply consists of reversing the input and output of the problem. A propagation model computes a series of ray pattern for a given set of sound-velocity profiles. Then, a multilayer Perceptron is taught to learn the inverse mapping i.e. the relation between the time pattern in input and the sound velocity profiles in output. Taking notation of 3.1. and introducing c for sound velocity in output and t for time in input, the cost function can be write as follows:

$$E(W) = \frac{1}{2} \sum_{m=1}^{N} \sum_{j=1}^{P} [c_{j,m}(W, t_m^0) - t_{j,m}^0]^2 \tag{6}$$

The optimal weight configuration W* is found by minimizing the cost function using a stochastic backpropagation algorithm [7]. After each presentation of an input-output pattern (step s), the weights are modified following a gradient rule:

$$w_{k,l}^s = w_{k,l}^{s-1} - \alpha \left(\frac{\partial E}{\partial w_{k,l}} \right)^s \tag{7}$$

with $\alpha > 0$.

3.2.2 Adjoint inverse modeling

The general scheme of adjoint inverse modeling (AIM) is given in figure 2. This approach consists of approximating the forward function with a multilayer Perceptrons. i.e. the relation between the sound velocity profiles and the time pattern. This is achieved following a standard backpropagation algorithm. The cost function to minimize becomes:

$$E(W) = \frac{1}{2} \sum_{m=1}^{N} \sum_{j=1}^{P} [t_{j,m}(W, c_m^0) - c_{j,m}^0]^2 \tag{8}$$

The optimal weight configuration $W*$ is computed following rule (7). Once the network in trained on the forward problem, the weights of the network are frozen. An initial solution is presented in input of the network. The network computes its ouputs which are an estimate of the temporal pattern associated to the initial sound velocity. The output error is backpropagated in the network and the initial solution is iteratively updated so that the output error decreases. The input adaptation is done following a gradient rule:

$$w_{k,l}^{s} = w_{k,l}^{s-1} - \alpha \left(\frac{\partial E}{\partial x_l}\right)^{s} \quad (9)$$

Note that the goal is to adjust the input (backpropagation-to-input) but weights remain unchanged. Such a scheme can be referred to "network inversion" [11]. The term "adjoint" inversion needs to be clarified. Given a model (in our case a ray tracing model), the purpose of the adjoint model is to give a convenient solution to compute the sensitivities of the output parameters of the model with respect to the input parameters. In words, the adjoint inversion model has to solve the problem: for a given perturbation of the output arount a target value (so, a local error), what is the perturbation to add on the inputs to minimize the error?

3.2.3 Distal inverse modeling

The distal inverse modeling (DIM) is a combination of the two previous ones. It has been proposed by Jordan and Rumelhart [12] for control inverse problems. Basically, it consists of composing both the inverse and the forward problem to retrieve the identity function. The forward problem is learned by a multilayer Perceptrons out of context. The weights are frozen. The distal scheme is used in coupling two nets. The time patterns to be inverted are presented to a randomly initialized net. The outputs of this network (which give a first estimate of the sound velocitiy profile) are presented to the forward network which computes an estimation of the temporal patterns. The error (i.e the difference between the estimation of the time patterns -outputs of the system- and the real time patterns -inputs of the system-) is backpropagated in the weight-frozen network (backpropagation-to-input) and consecutively backpropagated in the first network (backpropagation-to-weight). When the iteration scheme is completed, the first network achieves an estimation of the inverse problem. The general scheme is given in figure 3.

3.2.4 Remarks

The simplest approach to invert data is to reverse the role of inputs and outputs. This is basically the idea of the direct inverse modeling algorithm. For bijective inverse functions, one can always find a network which realizes the approximation on a given compact subset. The problem stems from the fact that inverse functions are often non univoque (e.g. inverse of constant functions). The philosophy of the adjoint inversion technique is to answer the following question: Given a frozen network, what input must be presented to the network to produce a set of target output values? This method does not enable to approximate multivoque functions but enables to control the solution (or to choose one among all solutions) respect to given "constraints" (assimilation of observations, initial guess, data assimilation,...). In addition, the adjoint inversion can be "goal-directed" in the research of the solution since it relies on a local estimate of a cost function and an iterative perturbation of an initial guess whereas the direct inverse modeling is a global searching method. As far as the distal inversion scheme is concerned, it is a combination of the two preceeding schemes. Its major advantage is that it enables to approximate multivoque functions (several output possible for a given input) by automatically choosing one among all solutions. It is goal directed and is convenient in case of "many-to-one" mapping, in particular in case of non convex domain for wich a linear combination of possible solutions (as the average for example) may not be acceptable.

4 RESULTS

4.1 Numerical experiment

We simulate a tomography experiment in the Western Mediterranean Sea. A source is moored at 100 meters depth and a receiver is moored at 400 meters depth (figure 4). We simulate the propagation on the abyssal plain (flat bottom) for a 50 kilometers propagation range. The forward problem is solved by a ray tracing model. An arbitrary set of sound velocity profiles $c(z)$ has been generated from typical situations in the Mediterranean Sea taken from historical data). The profiles are defined by 4 points at the respective depths of 0, 60, 300 and 2500 meters. The sound velocity variables are c(0), c(60), c and c(300). The sound velocity at the bottom is assumed to be constant (the oceanic variability is negligible). Each sound velocity values can vary in a given interval whose extension is assumed from oceanographic consideration. The set of sound velocity profiles is uniformly distributed in S=[1506.6,1508.6] x[1506.6,1507.6] x [1511.3,1511.8] (units in m/s). The increments in the intervals were respectively 0.2, 0.2, 0.1(m/s) for the learning base and 0.15, 0.07, 0,08 (m/s) for the testing base. Six identified and resolved rays were used for inversion. The profiles for wich the ray tracing algorithm did not predict the six rays have been arbitrarly removed (to avoid tedious manual refined computation). At this step, 1162 profiles were left. Among this set, 282 profiles were used to build the learning base. The performance of the inverse methods were tested on the 880 remaining profiles and are given in table 1. An example of profile inversion is given in figure 5 in the case of direct inverse modeling. A comparison is given with a usual linear method based upon the canonical decomposition of the sound velocity profiles on a basis of triangular functions.

4.2 Discussion

The performance of the three neural methods is good. A good accuracy is obtained (relative error lower than 3 p.c.). The absolute error is greater on the surface parameter which is expected to be less influent on the temporal pattern. Nevertheless, the relative error is greater on the shallow parameter (sound velocity at 60 meters). All neural methods compare with favor to the linear scheme. The performance of the linear scheme is maximum near the reference profile. The results get worse when increasing the "distance" to the reference profile. In order to keep the linear assumption we have restricted the sound velocity perturbation values. Even in favorable cases for linear inversions, all neural methods are more accurate (figure 6, table 1) We have compared the linear method and the direct inverse method for other types of mediterranean profiles [13]. Conversely to linear schemes, the neural performances do not depend on the distance to a reference profile. Compared together, the performances of the three neural methods are of the same higher of magnitude even though the direct approach seems to give better results. In the example presented in this work there is no advantage to work with one or another method (except for the computation time, which is smaller in the direct approach). The reason is that it is a well-posed problem with a quite regular function. Future work will be concerned with ill-posed problem (weakly influent parameters, non-univocity of the function,...).

5 CONCLUSION

Three neural methods to invert tomographic data have been proposed in this paper. All rely on the ability of Multilayered Perceptrons to approximate multidimensional functions. The inverse mapping between sound-velocity profiles and corresponding time patterns is learned by a neural net on the basis of a given set of behavior examples. Then, the performance in generalization (ability to retrieve sound velocity profiles from unknown time patterns) is used to perform inversions. The results show the efficiency of the neural approach which gives a very accurate retrieval of the sound velocity profiles. Furthermore, the methods compare with favor to a usual linear schemes. As it stands, our approach is restricted to range-independent media and would have to be extended to all media. The main problem is an exponential increase of the number of parameters to be estimated and so an increase of the number of examples in the learning base. The neural approach may appear very useful to invert the unresolved part of the signal. This is particularly interesting in double SOFAR channel environments [14] where ray identification and estimation is difficult or even impossible.

References

[1] Munk, W. H. and Wunsch, C. (1979) "Ocean acoustic tomography: a scheme for large scale monitoring", Deep-Sea Research, Vol. 26A, pp 123-161.

[2] Stephan, Y and Thiria, S. (1994) "Neural inversion for ocean acoustic tomography", in Inverse Problems in engineering Mechanics, Edited by Bui, Tanaka and al. Balkema Publishers, pp 55-60.

[3] Howe, B.M., Worcester, P.F. and Spindel, R.C. (1987) "Ocean acoustic tomography: mesoscale velocity", Journal of Geophysical research, Vol. 92, pp 3785-3805.

[4] Gaillard, F. (1992) "Evaluating the content of tomographic data: Application to mesoscale observations", Journal of Geophysical Research, Vol. 97, no C10, pp 15,490-15,505.

[5] Stephan, Y. and Martin-Lauzer, F.R. (1994) "GASTOM 90 tomography experiment. Data inversion", Proceedings of the Int. Conf. on Acoustics, Signal and Speech Processing, Vol 2, pp 301-304.

[6] Poggio, T. and Girosi, F. (1990) "Networks for approximation and learning", Proc. IEEE, Vol. 78,no 9.

[7] Rumelhart, D.E, Hinton, G.E. and Williams, R.J. (1986) "Learning representations by back-propagating errors", Nature, Vol 323, pp 533-536.

[8] Cybenko, G. (1989) "Approximation by superposition of sigmoidal function", Math. Control Signal Systems, Vol. 2, pp 303-314.

[9] Funahashi, K.I. (1989) "On the approximate realization of continuous mappings by neural networks", Neural Networks, Vol. 2, pp 183-192.

[10] Hornik, K., Stinchcomb, M., and White, H. (1989) "Multi-layer feedforward networks are universal approximators," Neural Networks, Vol. 2, pp 359-366.

[11] Linden, A. and Kinderman, J. (1989) "Inversion of multilayer nets", Proceedings of the International Joint Conference on Neural Networks, Vol II., pp 425-430.

[12] Jordan, M. and Rumelhart, D. (1992) "Forward models: supervised learning with a distal teacher", Cognitive Science, Vol. 16, pp 307-354.

[13] Stéphan, Y., Thiria, S. and Badran, F. (1995) "Inverting tomographic data with neural nets", Proceedings of OCEANS'95, San Diego.

[14] Stéphan, Y., Evennou, F. and Martin-Lauzer, F.R. (1995) "Acoustic Modeling and measurements in the Bay of Biscay in a double SOFAR channel", in Full Field Inversion Methods in Ocean and Seismo-Acoustics, O. Diachok et Al. (eds), Kluwer Academic Publishers.

Method	error	c(0)	c(60)	c(300)
DIM	mean error (m/s)	0.151	0.032	0.003
	standart deviation	0.113	0.021	0.002
	relative mean error (p.c.)	0.52	2.28	0.30
AIM	mean error (m/s)	0.184	0.040	0.002
	standart deviation	0.127	0.028	0.001
	relative mean error (p.c.)	0.83	2.86	0.20
DLM	mean error (m/s)	0.171	0.040	0.002
	standart deviation	0.126	0.029	0.001
	relative mean error (p.c.)	0.77	2.86	0.02
LM	mean error (m/s)	0.512	0.200	0.032
	standart deviation	0.372	0.144	0.022
	relative mean error (p.c.)	2.33	14.86	3.2

Table 1: Results of the Direct Inverse Method (DIM), the Adjoint Inverse Method (AIM), the DIstal Learning Method (DLM) and a Linear Method (LM) for 880 Mediterranean profiles.

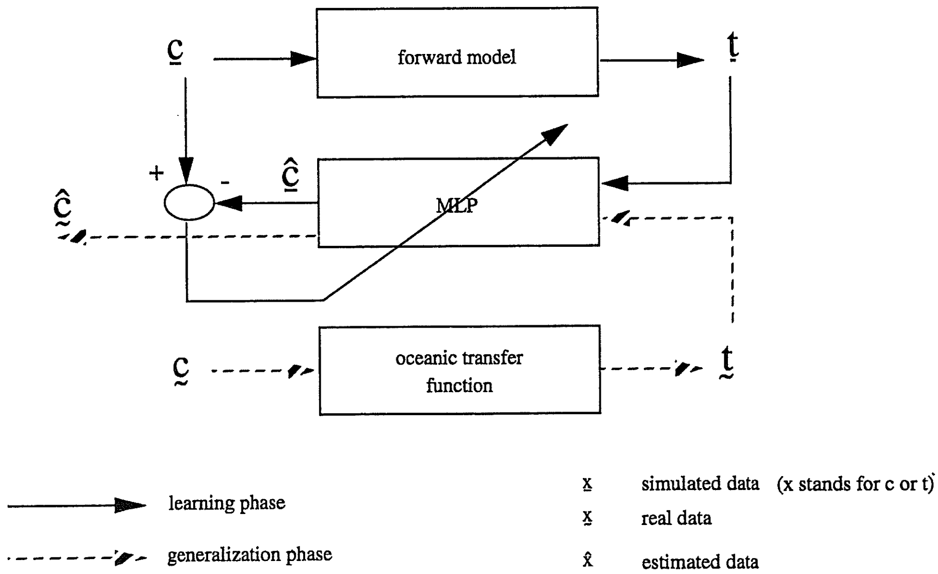

Figure 1: General scheme for neural direct inversion.

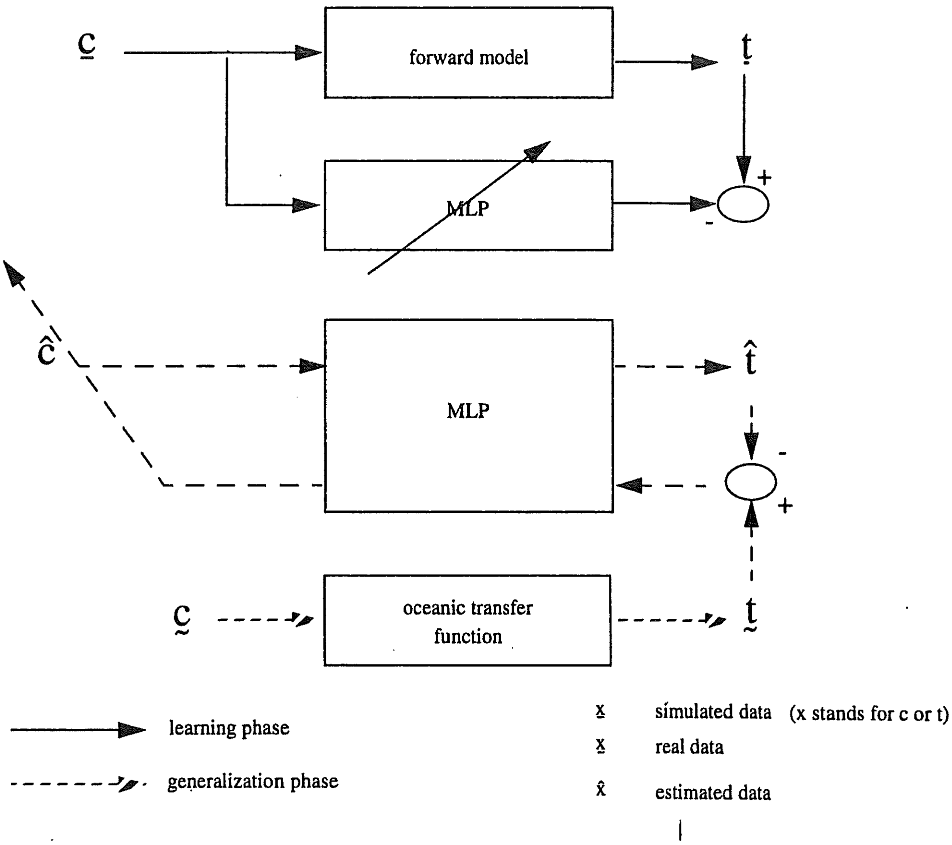

Figure 2: General scheme for neural adjoint inversion.

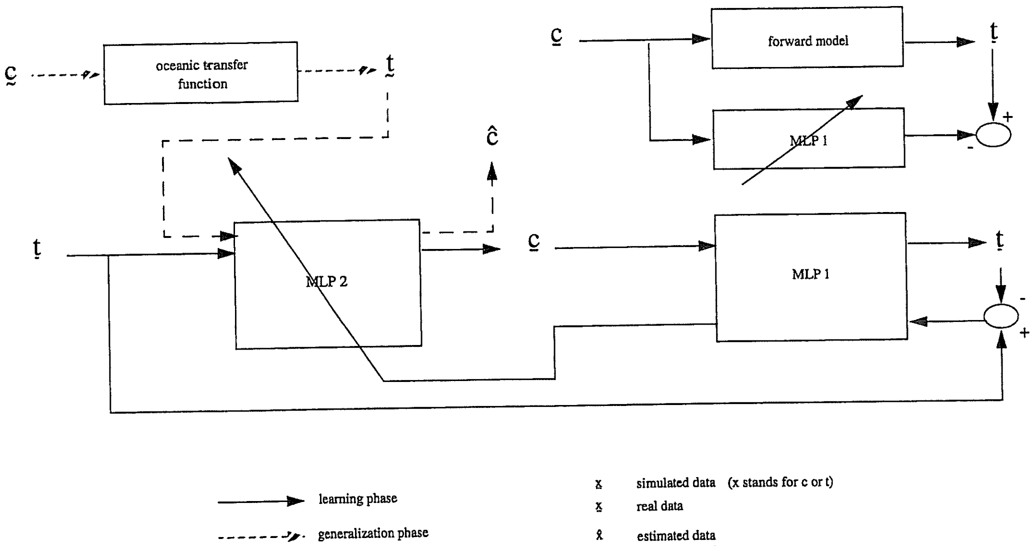

Figure 3: General scheme for neural distal inversion.

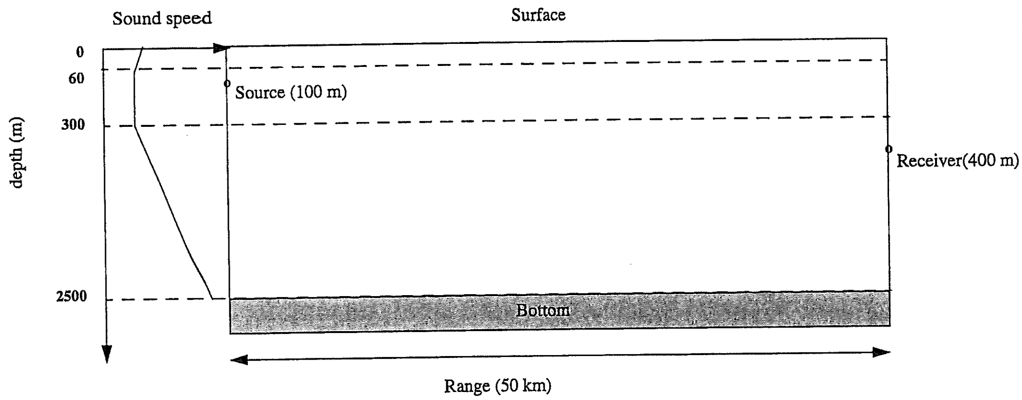

Figure 4: Design of the experiment for simulations.

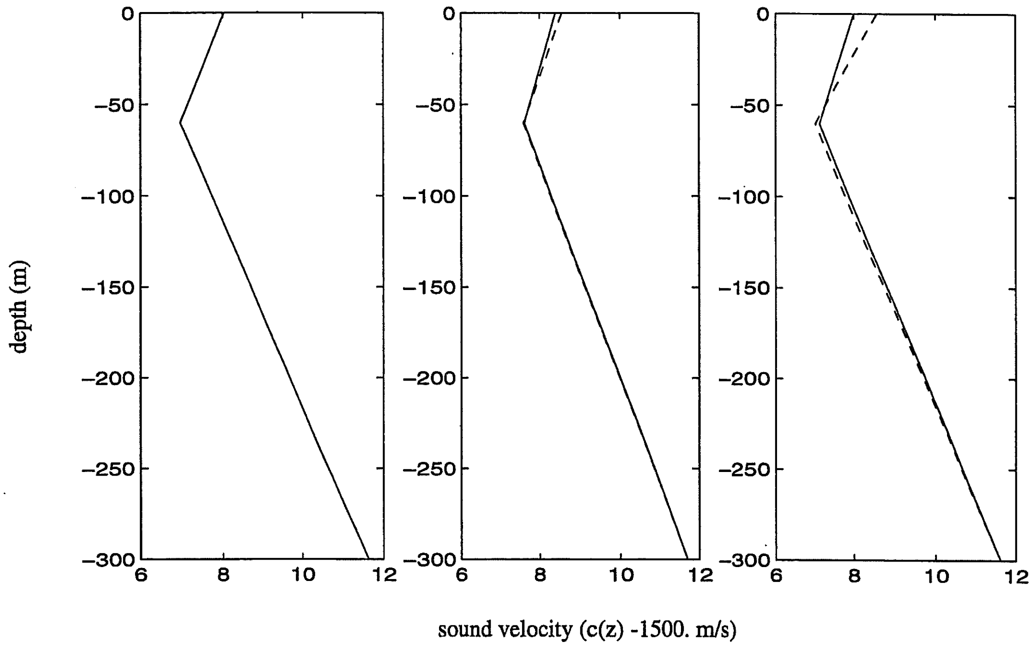

Figure 5: Results of the neural direct inversions on three characteristic profiles:
Left: Best approximated profile,
Middle: Inverted profile for which the error is closest to the mean error on the whole test set,
Right: Worse inverted profile.

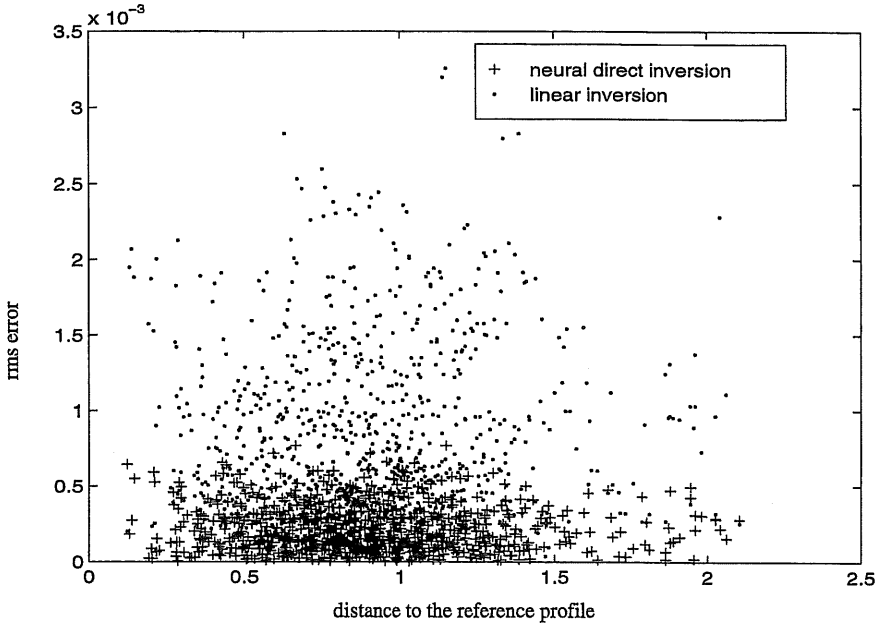

Figure 6: Comparisons between the neural Direct Inverse Method (DIM) and a Linear method(LM):
RMS error versus the distance to the reference profile (used for the linear scheme).