Theoretical and Computational Acoustics
D. Lee, Y-H Pao, M.H. Schultz, and Y-C Teng (Editors)

Backscatter Calculations in Simple Waveguide Geometries using Stepwise Coupled Normal Modes

Finn B. Jensen
SACLANT Undersea Research Centre
Viale San Bartolomeo 400, 19138 La Spezia, Italy

Abstract: Accurate numerical solutions for backscattering in a Pekeris waveguide with a single-facet protrusion on the seafloor are presented. In particular the effect of height and slope of the facet on the strength of the backscattered field is addressed. Moreover, two backscatter results are proposed as benchmark solutions for testing general-purpose reverberation codes.

1. Introduction

It is believed that strong backscatter in ocean waveguides is always associated with large surfaces (facets) with optimal orientation relative to the incident sound. To try to verify this hypothesis, we here compute backscattering from a single-facet protrusion on the seafloor in a shallow-water waveguide. In particular, we address the effect of height and slope of the facet on the strength of the backscattered field. The simulations are carried out with a 'certified' benchmark code, which has been recently tested on a set of canonical low-frequency reverberation problems [1,2].

2. Test Problem Description

We consider the simplest possible test environment (Fig. 1) consisting of a 200-m deep shallow-water waveguide bounded above by a pressure-release surface and below by a penetrable, homogeneous fluid bottom. The water column is isovelocity with $c_W = 1500\,\mathrm{m/s}$. The bottom properties are $c_B = 1700\,\mathrm{m/s}$, $\alpha_B = 0.5\,\mathrm{dB}/\lambda$, and $\rho_B = 1.5\,\mathrm{g/cm^3}$. We consider a 2D problem with translational symmetry in the y-direction. The obstacle is a simple step protrusion on the bottom placed 1.5 km downrange and having the same acoustic properties as the seabed. Thus there is only one scattering facet, namely the front-end of the protrusion. The source is a 300-Hz Gaussian beam directed towards the scattering facet and given by

$$\psi(0,z) = \sqrt{k_0}\,\tan\theta_1\, e^{-\frac{k_0^2}{2}(z-z_s)^2\tan^2\theta_1}\, e^{ik_0(z-z_s)\sin\theta_2}\,, \tag{1}$$

where $\theta_1 = 2°$ is the halfwidth of the source aperture, and $\theta_2 = 3.15°$ is the beam tilt with respect to the horizontal, measured positive downward. As shown in Fig. 2, this beam provides a uniform insonification of the obstacle for heights up to approximately 35 m $(7\,\lambda)$.

3. The Acoustic Model

While numerical codes for solving propagation problems in ocean acoustics are abundant [3], there is much less choice when dealing with scattering, particularly backscattering. Our choice was the two-way coupled mode model (COUPLE) developed by Evans some years ago [4]. This code was recently updated to include a 'sponge' layer deep in the bottom [5], thus improving the computational performance by a factor 20–50. COUPLE has previously been used for benchmarking scattering problems in range-dependent ocean waveguides [1,2], and is considered a reference code for this type of work.

The solution technique is based on a range-discretization of the environment into segments with range-invariant properties, but with allowance for arbitrary variations of sound speed, attenuation, and density with depth. Hence, a bottom feature of the type shown in Fig. 1 would have its front surface sliced into a number of range segments with slightly changing water depth in each segment. This stair-step approximation approaches the continuously varying bottom slope for an increasing number of range segments [6]. After the range discretization (around 10 steps/wavelength), a full-spectrum mode set is computed for each segment. Finally, by imposing appropriate boundary conditions between segments together with a known source condition at range zero and a radiation condition at infinity, a solution for the acoustic field based on propagator matrices can be constructed. Presently, the coupled-mode code has been set up for fluid media only.

4. Numerical Results

Before presenting the backscatter solutions for the single-facet bottom protrusion in Fig. 1, let us briefly address the selection of model parameters that ensure accurate and numerically stable answers. After a series of convergence tests, we settled on the following set of parameters: A computational depth domain of $H_B = 300\,\mathrm{m}$, extending from the sea surface to a 'false' rigid bottom 100 m down in the sediment; a 50-m thick sponge layer starting at depth $H_A = 250\,\mathrm{m}$ and with and attenuation that increases linearly from $0.5\,\mathrm{dB}/\lambda$ at the top of the layer to $10\,\mathrm{dB}/\lambda$ at the bottom of the layer. The number of modes needed to provide a full-spectrum solution was found to be NM = 120, whereas the stair-step sampling of the sloping protrusion was taken to be $\Delta x = .08\,\lambda$ [6]. The computational effort on a DEC 3000/400 workstation ranged from a few minutes for a vertical facet (2 mode sets only) to 3.5 hours for a 30° facet of length $12\,\lambda$ and involving the computation and coupling of 150 mode sets, with 120 modes in each set.

4.1 EFFECT OF FACET HEIGHT

As a measure of the scattering strength we use the mean intensity over depth (0–200 m) of the back-propagated field at the source range. The result for a vertical protrusion of varying height is shown in Fig. 3(a). Note that the backscatter level is nearly constant ($\sim 48\,\mathrm{dB}$) for obstacles larger than $5\,\lambda$, whereas the level falls off rapidly for smaller obstacle heights.

Full field plots of the backscattered energy for different facet heights are given in Fig. 4. For large vertical facets ($h \geq 5\,\lambda$) energy is scattered right back towards the source as a well-defined beam, see upper panel. For decreasing facet height, two things occur: The backscattered beam weakens in intensity and at the same time it is shifted to steeper propagation angles, thus causing increased bottom loss for the back-propagated energy originating from small facets. These results are not surprising, but the numerical simulations allow a quantification of the effect. For instance, for this particular environment [see Fig. 3(a)], a vertical facet of $0.1\,\lambda$ height is seen to have a 40 dB lower backscatter strength than a facet of $5\,\lambda$ height or larger.

4.2 EFFECT OF FACET SLOPE

We next take a $7\,\lambda$ protrusion and vary the slope of the front surface between 30 and 90°. The result for the mean backscatter is shown in Fig. 3(b) and we note a couple of interesting features. First, slopes above 75° give strong backscatter, which, as shown in the upper two panels of Fig. 5, corresponds to a situation where the backscattered beam interacts with the seabed within the critical angle [$\theta_{cr} = \arccos(1500/1700) \simeq 28°$], and hence suffer little bottom loss. This is followed by decreasing backscatter strength with decreasing facet slope, caused mainly by an increase in bottom loss for the steeper-angled reflected beams, see panel 3 in Fig. 5. The second interesting feature is the strong backscatter around 45°, which, as shown in the lower panel of Fig. 5, corresponds to a path reflected off the facet directly towards the sea surface and then back again. This important path is difficult to compute in many reverberation models since it includes both horizontally and vertically propagating energy.

4.3 BENCHMARK SOLUTIONS

The generation of reference solution for backscattering in ocean waveguides is of high priority for testing general-purpose numerical models. A concerted effort to obtain such benchmark solutions was recently undertaken [1,2] but further results are desirable.

We here propose the results given in Fig. 6, which displays the backscattered field strength over depth at the source range for two different facet slopes of 45° and 90°. The facet height is in both cases $7\,\lambda$ and the environmental parameters are as given in Fig. 1. Also displayed in Fig. 6 is the Gaussian source field given by Eq. (1). The peak pressure in the source field is computed as $\mathrm{TL} = -20\,\log|\psi(0, z_0)| = 28.15\,\mathrm{dB}$. One of the two field solutions (90° slope) involves mainly horizontally propagating energy and is therefore ideal for checking two-way codes that invoke single-scatter and small-angle assumptions. The second case (45° slope) is computationally much more difficult. Since this case involves both horizontally and vertically propagating energy it requires a full-spectrum solution approach as provided by the COUPLE code.

5. Summary and Conclusions

It is clear from this study that strong backscattering is associated with large facets ($h \geq 5\lambda$) and steep slopes ($\theta > 75°$). However, other slopes may also give significant contributions, e.g. the 45° slope, which constitutes the ultimate test problem for backscatter models. The computational effort with the coupled-mode code used here is considerable, and there clearly is a need to develop more efficient scattering models that can be applied at higher frequencies as well as in deeper water.

References

[1] F.B. Jensen and P. Gerstoft, "Benchmark solutions for backscattering in simple waveguide geometries," in *Proceedings of the second European Conference on Underwater Acoustics, Vol. I*, edited by L. Bjørnø (European Commission, Luxembourg, 1994), pp. 119–124.

[2] S.A. Chin-Bing, D.B. King, J.A. Davis and R.B. Evans, eds., *R&S Workshop: Proceedings of the Reverberation and Scattering Workshop* (Naval Research Laboratory, Stennis Space Center, MS, 1995).

[3] F.B. Jensen, W.A. Kuperman, M.B. Porter, and H. Schmidt, *Computational Ocean Acoustics* (Amer. Inst. of Physics, New York, 1994).

[4] R.B. Evans, "A coupled mode solution for acoustic propagation in a waveguide with stepwise depth variations of a penetrable bottom," J. Acoust. Soc. Amer. **74**, 188–195 (1983).

[5] R.B. Evans, "A reverberation calculation using stepwise coupled modes," (in Ref. [2]).

[6] F.B. Jensen, "On the use of stair steps to approximate bathymetry changes in ocean waveguides," (these proceedings)

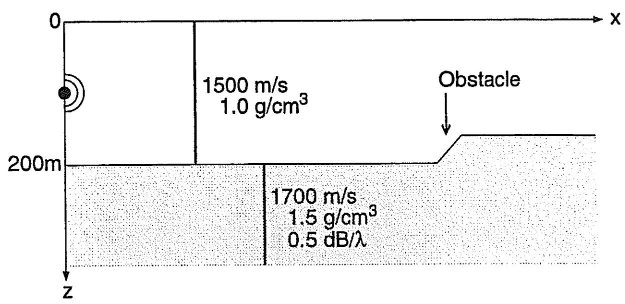

Figure 1: Geometry for backscatter calculations.

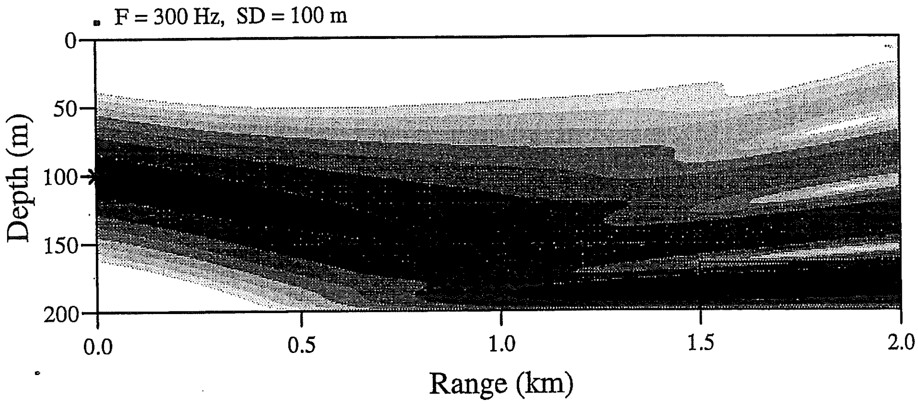

Figure 2: Beam insonification of obstacle placed on the bottom 1.5 km from the source. The contour levels (from black to white) are 30 to 60 dB in steps of 5 dB.

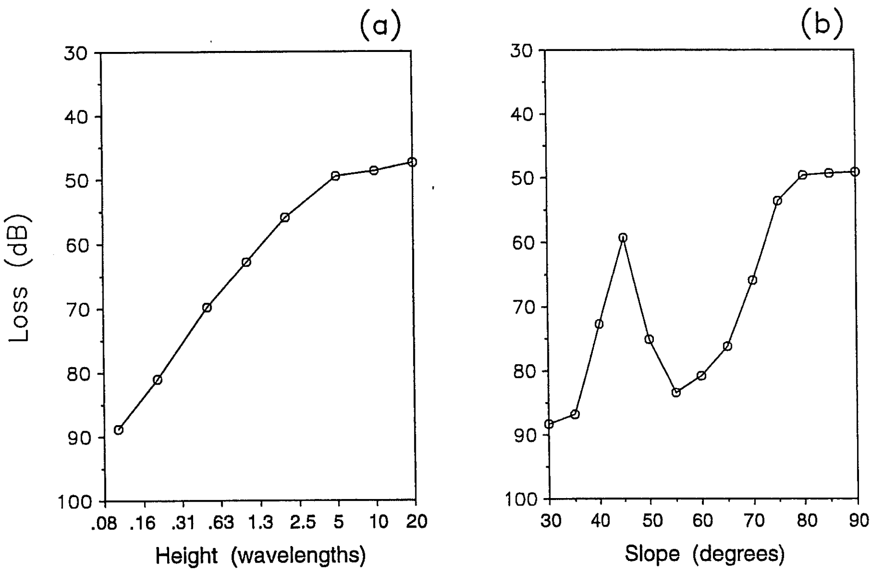

Figure 3: Mean backscatter level as a function of (a) obstacle height and (b) obstacle slope.

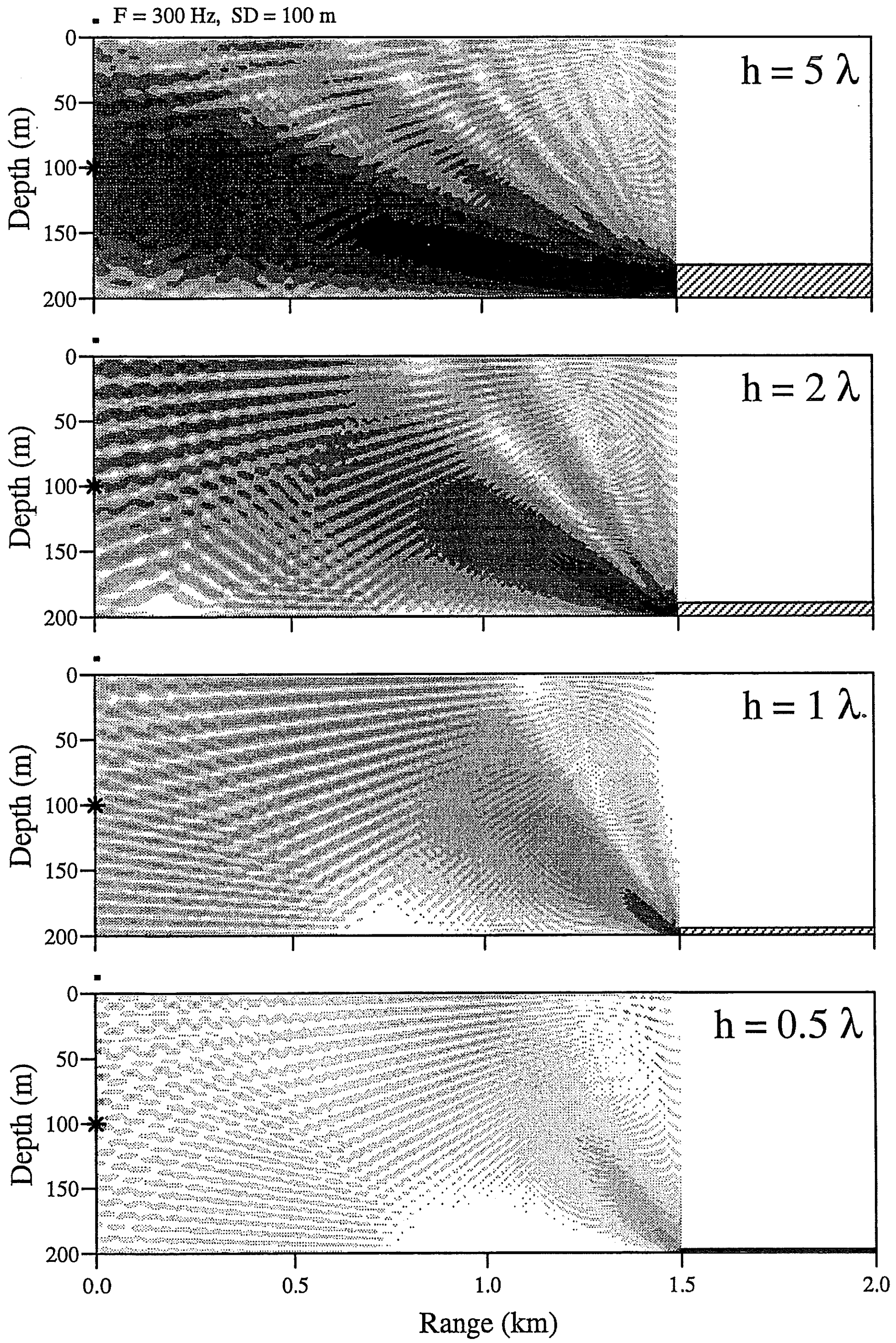

Figure 4: Backscattered field for different heights of a vertical bottom facet. The contour levels (from black to white) are 40 to 70 dB in steps of 5 dB.

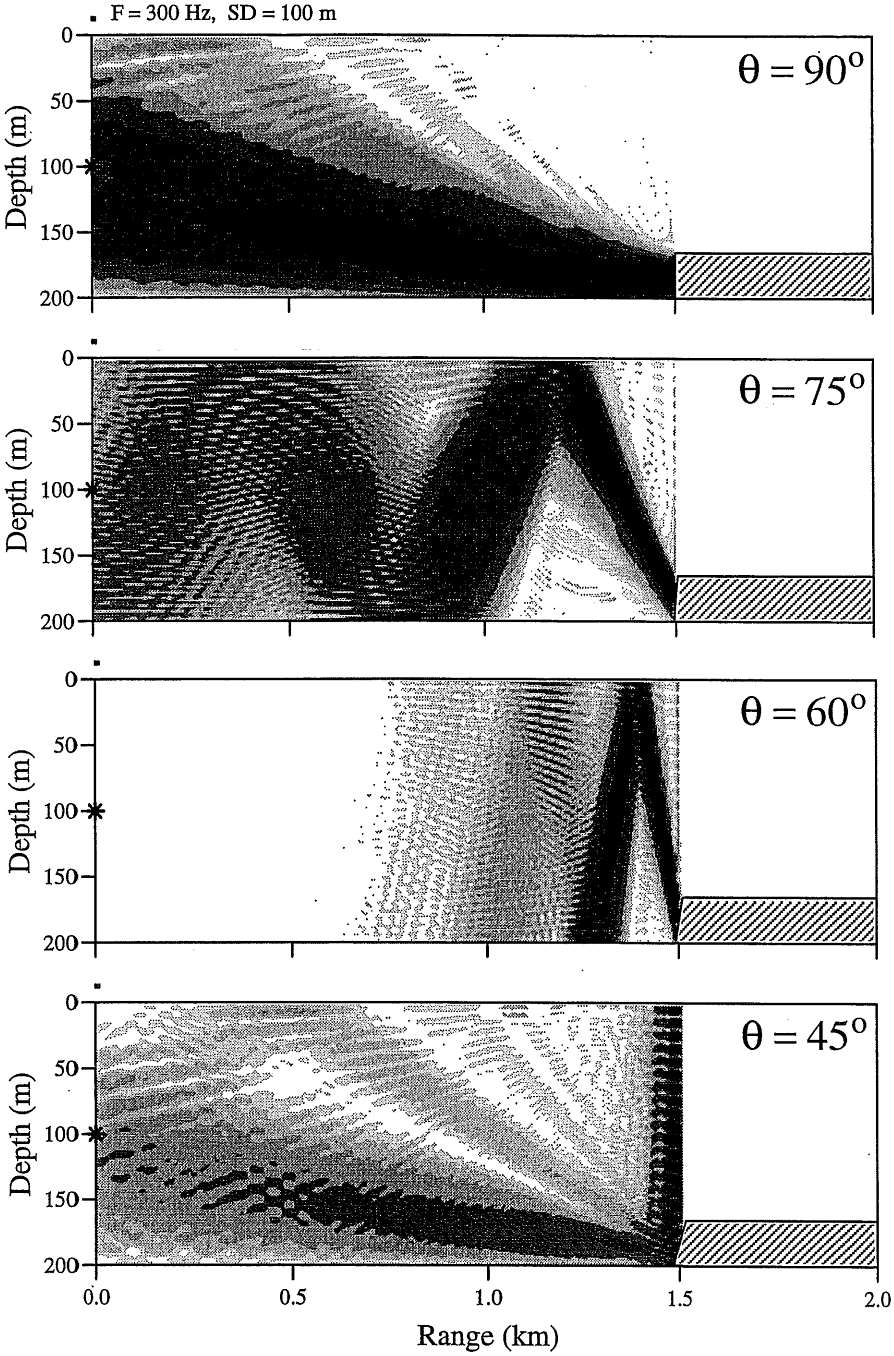

Figure 5: Backscattered field for different slopes of the 35-m high bottom facet. The contour levels (from black to white) are 40 to 70 dB in steps of 5 dB.

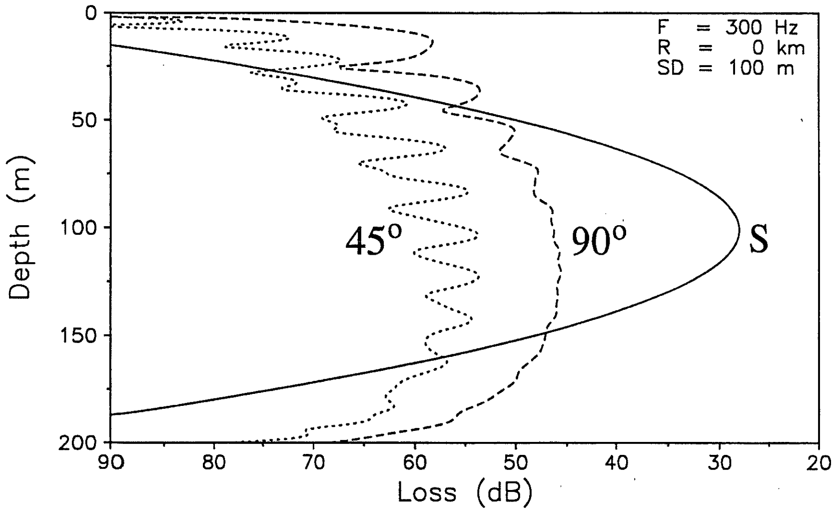

Figure 6: Benchmark solutions for backscattered field from a 7-λ high bottom facet. Results are given for both a 45° and a 90° facet slope. The heavy line marked 'S' is the source field.