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# NUMERICAL STUDIES OF CONJUGATED INFINITE ELEMENTS FOR ACOUSTICAL RADIATION

R. J. ASTLEY and J. A. HAMILTON

*Department of Mechanical Engineering,*
*University of Canterbury,*
*Private Bag 4800, Christchurch, New Zealand*

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Aspects of conjugated infinite element schemes for unbounded wave problems are reviewed and a general formulation is presented for elements of variable order based on separable shape functions expressed in terms of prolate and oblate spheroidal coordinates. The formulation encompasses both "conjugated Burnett" and "Astley–Leis" elements. The performance of the two approaches is compared for steady multipole wave fields and the effect of the radial basis on the condition number of the resulting equations is discussed. Transient formulations based on these elements are derived and methods for solving the resulting transient equations are discussed. The use of an implicit time stepping scheme coupled with an indirect iterative solver is shown to give fast transient solutions which do not require matrix inversion.

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# A REFLECTION FREE BOUNDARY CONDITION FOR PROPAGATION IN UNIFORM FLOW USING MAPPED INFINITE WAVE ENVELOPE ELEMENTS

WALTER EVERSMAN

*University of Missouri-Rolla, 1870 Miner Circle, Rolla, MO, 65401, USA*

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Variable order mapped infinite wave envelope elements are developed for finite element modeling of acoustic radiation in a uniformly moving medium. These elements are used as a nonreflecting boundary condition for computations on an infinite domain in which a radiating body is immersed in a moving medium which is essentially undisturbed outside of the near field. The mapped elements provide a boundary condition equivalent to element stiffness, mass, and damping matrices appended to an inner standard FEM mesh. A demonstration of the performance of mapped elements as influenced by element order is given in the context of acoustic radiation from a turbofan inlet and exhaust.

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# A REVIEW OF INFINITE ELEMENT METHODS FOR EXTERIOR HELMHOLTZ PROBLEMS

KLAUS GERDES

*Department of Mathematics, Chalmers University of Technology, SE-412 96 Gothenburg, Sweden*

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This work is devoted to review infinite element discretizations for the Helmholtz equation in exterior domains, which have become popular in recent years, as many research papers on this topic have appeared in the literature. The early contributions were mostly motivated by engineering considerations and the variational formulations have, in general, not been stated in a mathematically precise way. Only recently, theoretical aspects of the infinite element methodology have been analyzed and helped to put the different formulations into a mathematical framework. We build upon this, and present and compare the infinite element formulations within this context.

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# ON FUNDAMENTAL ASPECTS OF EXTERIOR APPROXIMATIONS WITH INFINITE ELEMENTS

FRANK IHLENBURG

*Germanischer Lloyd, Vorsetzen 32, D-20459 Hamburg, Germany*

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Over the last years, infinite elements have become a popular method of exterior approximation in scattering problems. Addressing the scalar case (Helmholtz equation), this paper gives a state-of-the-art review on the convergence theory for the approximation used in radial infinite elements. The focus is on the stability of different variational formulations that were proposed in the context of this approximation. Exterior approximation on partitions with finite and infinite elements is also considered. The review is preceded by an outline of the relevant general theory. Open questions are listed in the conclusion of the presentation.

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# FINITE ELEMENT SOLUTION OF TWO-DIMENSIONAL ACOUSTIC SCATTERING PROBLEMS USING ARBITRARILY SHAPED CONVEX ARTIFICIAL BOUNDARIES

RABIA DJELLOULI, CHARBEL FARHAT,
ANTONINI MACEDO, and RADEK TEZAUR

*Department of Aerospace Engineering Sciences and Center for Aerospace Structures,
University of Colorado at Boulder, Boulder, CO 80309-0429, USA*

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For elongated scatterers such as submarines, we show that the generalization of the Bayliss–Turkel nonreflecting boundary conditions to arbitrarily shaped convex artificial boundaries improves significantly the computational efficiency of finite element methods for the solution of acoustic scattering problems.

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# THIRD-ORDER DOUBLY ASYMPTOTIC APPROXIMATIONS FOR COMPUTATIONAL ACOUSTICS

THOMAS L. GEERS and BRADY J. TOOTHAKER
*Center for Acoustics, Mechanics and Materials,*
*Department of Mechanical Engineering,*
*University of Colorado, Boulder, CO 80309-0427, USA*

Received 9 July 1999
Revised 3 November 1999

A fully and symmetrically matched, third-order doubly asymptotic computational boundary ($DAA_3$) is presented in both operator and matrix forms. Its performance is then compared with that of other DAA boundaries for the spherical geometry, which admits an exact series solution. Based on stability, accuracy and passivity as performance criteria, $DAA_3$ is found to be superior to all other DAAs formulated to date. However, there are other third-order forms that are nearly as good, and are easier to implement in boundary-element form. The existence of both $DAA_2$ and $DAA_3$ boundaries enables the analyst to assess solution accuracy for a given problem through the comparison of results produced with these higher approximations of different order.

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# ANALYTICAL AND NUMERICAL STUDIES OF A FINITE ELEMENT PML FOR THE HELMHOLTZ EQUATION

ISAAC HARARI and MICHAEL SLAVUTIN
*Department of Solid Mechanics, Materials and Structures,*
*Tel Aviv University, 69978 Ramat Aviv, Israel*

ELI TURKEL
*School of Mathematical Sciences*
*Tel Aviv University, 69978 Ramat Aviv, Israel*

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A symmetric PML formulation that is suitable for finite element computation of time-harmonic acoustic waves in exterior domains is analyzed. Dispersion analysis displays the dependence of the discrete representation of the PML parameters on mesh refinement. Stabilization by modification of the coefficients is employed to improve PML performance, in conjunction with standard stabilized finite elements in the Helmholtz region. Numerical results validate the good performance of this finite element PML approach.

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# CONTINUED-FRACTION ABSORBING BOUNDARY CONDITIONS FOR THE WAVE EQUATION

MURTHY N. GUDDATI

*Department of Civil Engineering, North Carolina State University,
Campus Box 7908, Raleigh, NC 27695-7908, USA*

JOHN L. TASSOULAS

*Department of Civil Engineering, The University of Texas at Austin,
Austin, TX 78712, USA*

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Absorbing boundary conditions are generally required for numerical modeling of wave phenomena in unbounded domains. Local absorbing boundary conditions are generally preferred for transient analysis because of their computational efficiency. However, their accuracy is severely limited because the more accurate high-order boundary conditions cannot be implemented easily. In this paper, a new arbitrarily high-order absorbing boundary condition based on continued fraction approximation is presented. Unlike the existing boundary conditions, this one does not contain high-order derivatives, thus making it amenable to implementation in conventional $C^0$ finite element and finite difference methods. The superior numerical properties and implementation aspects of this boundary condition are discussed. Numerical examples are presented to illustrate the performance of these new high-order boundary condition.

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# OPTIMAL LOCAL NONREFLECTING BOUNDARY CONDITIONS FOR TIME-DEPENDENT WAVES

IGOR PATLASHENKO

*Hummingbird Communications LTD., 1 Sparks Avenue,
North York, Ontario, Canada M2H 2W1*

DAN GIVOLI

*Department of Aerospace Engineering, and Asher Center for Space Research,
Technion — Israel Institute of Technology, Haifa 32000, Israel*

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Nonreflecting Boundary Conditions (NRBCs) are often used on artificial boundaries as a method for the numerical solution of wave problems in unbounded domains. Recently, a two-parameter hierarchy of optimal local NRBCs of increasing order has been developed for elliptic problems, including the problem of time-harmonic acoustic waves. The optimality is in the sense that the local NRBC best approximates the exact non-local Dirichlet-to-Neumann (DtN) boundary condition in the $L_2$ norm for functions which can be Fourier-decomposed. The optimal NRBCs are combined with finite element discretization in the computational domain. Here this approach is extended to *time-dependent* acoustic waves. In doing this, the Semi-Discrete DtN approach is used as the starting point. Numerical examples involving propagating disturbances in two dimensions are given.

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# MIXED FINITE ELEMENTS WITH MASS-LUMPING FOR THE TRANSIENT WAVE EQUATION

GARY COHEN and SANDRINE FAUQUEUX

*INRIA, Domaine de Voluceau, Rocquencourt, B.P. 105,*
*78153 Le Chesnay Cedex, France*
*gary.cohen@inria.fr*
*sandrine.fauqueux@inria.fr*

Received 15 June 1999
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Solving the acoustics equation by finite elements with mass-lumping requires the use of spectral elements. Although avoiding the inversion of a mass-matrix at each time-step, these elements remain expensive from the point of view of the stiffness-matrix. In this paper, we give a mixed finite element method which provides a factorization of the stiffness-matrix which leads to a gain of storage and computation time which grows with the order of the method and the dimension in space. After proving the equivalence between classical spectral elements and this method, we give a dispersion analysis on nonregular periodic meshes. Then, we analyze the accuracy and the stability of $Q_3$ and $Q_5$ approximations on numerical tests in 2D.

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# SHORT WAVE MODELLING USING SPECIAL FINITE ELEMENTS

OMAR LAGHROUCHE and PETER BETTESS
*School of Engineering, University of Durham, Durham DH1 3LE, UK*

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The solutions to the Helmholtz equation in the plane are approximated by systems of plane waves. The aim is to develop finite elements capable of containing many wavelengths and therefore simulating problems with large wave numbers without refining the mesh to satisfy the traditional requirement of about ten nodal points per wavelength. At each node of the meshed domain, the wave potential is written as a combination of plane waves propagating in many possible directions. The resulting element matrices contain oscillatory functions and are evaluated using high order Gauss-Legendre integration. These finite elements are used to solve wave problems such as a diffracted potential from a cylinder. Many wavelengths are contained in a single finite element and the number of parameters in the problem is greatly reduced.

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# A NUMERICAL COMPARISON OF FINITE ELEMENT METHODS FOR THE HELMHOLTZ EQUATION

ASSAD A. OBERAI and PETER M. PINSKY

*Division of Mechanics and Computation, Department of Mechanical Engineering, Room 262, Durand Building, Stanford University, Stanford, CA 94305, USA*

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Three finite element formulations for the solution of the Helmholtz equation are considered. The performance of these methods is compared by performing a discrete dispersion analysis and by solving two canonical problems on nonuniform meshes. It is found that: (1) The scaled $L_2$ error for the Galerkin method, using linear interpolation functions, grows as $k(kh)^2$, indicating the pollution inherent in this method; (2) The Galerkin least squares method is more accurate, but does display significant pollution error; (3) The residual-based method of Oberai & Pinsky,[7] which was designed to be almost pollution-free for uniform meshes retains its accuracy on nonuniform meshes; (4) The computational cost of implementing all these formulations is approximately the same.

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# EFFICIENT COMPUTATION OF MULTI-FREQUENCY FAR-FIELD SOLUTIONS OF THE HELMHOLTZ EQUATION USING PADÉ APPROXIMATION

MANISH MALHOTRA* and PETER M. PINSKY
*Department of Mechanical Engineering, Stanford University, CA 94305–4040, USA*

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For many problems in exterior structural acoustics, the solution is required to be computed over multiple frequencies. For some classes of these problems, however, it may be sufficient to evaluate the multiple frequency solutions over restricted regions of the spatial domain. Examples include optimization and inverse problems based on the minimization of a functional defined over a specified surface or sub-region. For such problems, which include both near-field and far-field computations, we recently proposed an efficient algorithm to compute the partial-field solutions at multiple frequencies simultaneously. In this paper, we consider the particular case of far-field computations and simplify the recently proposed algorithm by exploiting the symmetry of linear operators. The approach involves a reformulation of the Dirichlet-to-Neumann (DtN) map based finite-element matrix problem into a transfer-function form that can efficiently describe the far-field solution. A multi-frequency approximation of the transfer function is developed by constructing matrix-valued Padé approximation of the transfer function via a symmetric, banded Lanczos process. Numerical tests illustrate the accuracy of the approach for a wide range of frequencies and cost reductions of an order of magnitude when compared to commonly used factorization based methods.

*Present address: Sun Microsystems, UMPK24-201, 901 San Antonio Road, Palo Alto 94303.

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# AN ITERATIVE TIME-STEPPING METHOD FOR SOLVING FIRST-ORDER TIME DEPENDENT PROBLEMS AND ITS APPLICATION TO THE WAVE EQUATION*

GÉZA SERIANI

*Istituto Nazionale di Oceanografia e di Geofisica Sperimentale*
*Borgo Grotta Gigante 42/C, I-34010 Trieste, Italy*
*gseriani@ogs.trieste.it*

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Equations describing dynamic problems, after spatial discretization by using the finite element or spectral element method, lead to solve large systems of ODE in time. A family of new time integration algorithms based on an iterative time-stepping (ITS) approach is proposed for solving these systems. The method is developed for first- and second-order differential equations, and applied, in particular, to wave equation. It is an implicit time marching method in which, at each time-step, the solution is computed by a fixed-point scheme. The analysis show that the method is accurate, unconditionally stable and that it allows for efficient and parallel implementations because no matrix inversion is required and only matrix-vector multiplications and vector scaling operations are involved.

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