



FICTITIOUS DOMAINS, MIXED FINITE ELEMENTS AND PERFECTLY MATCHED LAYERS FOR 2-D ELASTIC WAVE PROPAGATION*

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We design a new and efficient numerical method for the modelization of elastic wave propagation in domains with complex topographies. The main characteristic is the use of the fictitious domain method for taking into account the boundary condition on the topography: the elastodynamic problem is extended in a domain with simple geometry, which permits us to use a regular mesh. The free boundary condition is enforced introducing a Lagrange multiplier, defined on the boundary and discretized with a nonuniform boundary mesh. This leads us to consider the first-order velocity-stress formulation of the equations and particular mixed finite elements. These elements have three main nonstandard properties: they take into account the symmetry of the stress tensor, they are compatible with mass lumping techniques and lead to explicit time discretization schemes, and they can be coupled with the Perfectly Matched Layer technique for the modeling of unbounded domains. Our method permits us to model wave propagation in complex media such as anisotropic, heterogeneous media with complex topographies, as it will be illustrated by several numerical experiments.

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HIGHER-ORDER MASS-LUMPED FINITE ELEMENTS FOR THE WAVE EQUATION*

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The finite-element method (FEM) with mass lumping is an efficient scheme for modeling seismic wave propagation in the subsurface, especially in the presence of sharp velocity contrasts and rough topography. A number of numerical simulations for triangles are presented to illustrate the strength of the method. A comparison to the finite-difference method shows that the added complexity of the FEM is amply compensated by its superior accuracy, making the FEM the more efficient approach.

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NUMERICAL STUDY OF ELASTIC WAVE SCATTERING BY CRACKS OR INCLUSIONS USING THE BOUNDARY INTEGRAL EQUATION METHOD*

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In this paper, we use a 2-D elastodynamic boundary integral equation or boundary element method (BEM) to solve multiple scattering problems due to existence of cracks or inclusions. The method is based on the integral representation of a scattered wavefield by assuming a fictitious source distribution on the scattering objects or inclusions (i.e. mathematical description of Huygens' principle), and the fictitious source distribution can be found by matching appropriate boundary conditions at the boundary of the inclusions. The method is called indirect boundary element method. Three numerical examples are presented to demonstrate the versatility of the BEM method. The first example shows that different spatial arrangements of the same scatters lead to profound differences in scattering characteristics, in particular the frequency contents of the transmitted wavefields using the method of time-frequency analysis. The second example shows the effects of power-law or fractal distribution of scalelengths on transmitted wavefields, and we conclude that frequency characteristics, such as the frequency of the peak attenuation, can be related to spatial size parameters of the model. In the third example, we show that orientated inclusions with aspect ratio less than unity have strong effects on the amplitudes of transmitted waves, and this has an important implication in characterizing inclusions and fractures using azimuthal variation in amplitudes (or attenuation anisotropy).

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FINITE-DIFFERENCE MODELING IN MEDIA WITH MANY SMALL-SCALE CRACKS*

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We discuss a finite-difference modeling technique for scalar, two-dimensional wave propagation in a medium containing a large number of small-scale cracks. The embedding medium can be heterogeneous. The boundaries of the cracks are not represented in the finite-difference mesh but the cracks are incorporated as distributed point sources. This enables the use of grid cells that are considerably larger than the crack sizes. We compare our method to an accurate integral-equation solution for the case of a homogeneous embedding and conclude that the finite-difference technique is accurate and computationally fast.

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SCALE AND ANGLE DEPENDENT REFLECTION PROPERTIES OF SELF-SIMILAR INTERFACES*

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We propose an alternative parameterization of seismic reflectors in the subsurface, in terms of self-similar singularities, which are generalizations of stepfunctions. This parameterization captures the multi-scale behavior of real sonic P-wave velocity logs, as can be derived by performing modulus maxima analysis on wavelet-transformed well-logs. Results on synthetic seismic reflection data, modeled in real well-logs, show that a singularity parameter can be retrieved, that is consistent with the parameter derived directly from the well-log.

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ELASTIC WAVES GENERATED BY HIGH-SPEED TRAINS*

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High-speed trains can generate vibrations that propagate away from the track. We present an accurate and efficient method for computing the vibrations in an elastic half-space at both small and large distances from the track. The method takes possible oscillatory behavior of the train into account. We conclude that vibrations, generated by oscillating trains, can be observed at large distances from the track, even if the train speed is lower than the Rayleigh wave speed.

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ADAPTIVE FINITE ELEMENT TECHNIQUES FOR THE ACOUSTIC WAVE EQUATION*

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We present an adaptive finite element method for solving the acoustic wave equation. Using a global duality argument and Galerkin orthogonality, we derive an identity for the error with respect to an arbitrary functional output of the solution. The error identity is evaluated by solving the dual problem numerically. The resulting local cell-wise error indicators are used in the grid adaptation process. In this way, the space-time mesh can be tailored for the efficient computation of the quantity of interest. We give an overview of the implementation of the proposed method and illustrate its performance by several numerical examples.

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WAVE PROPAGATION IN 2-D ELASTIC MEDIA USING A SPECTRAL ELEMENT METHOD WITH TRIANGLES AND QUADRANGLES*

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We apply a spectral element method based upon a conforming mesh of quadrangles and triangles to the problem of 2-D elastic wave propagation. The method retains the advantages of classical spectral element methods based upon quadrangles only. It makes use of the classical Gauss-Lobatto-Legendre formulation on the quadrangles, while discretization on the triangles is based upon interpolation at the Fekete points. We obtain a global diagonal mass matrix which allows us to keep the explicit structure of classical spectral element solvers. We demonstrate the accuracy and efficiency of the method by comparing results obtained for pure quadrangle meshes with those obtained using mixed quadrangle-triangle and triangle-only meshes.

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A COMPARATIVE STUDY OF EXPLICIT DIFFERENTIAL OPERATORS ON ARBITRARY GRIDS*

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We compare explicit differential operators for unstructured grids and their accuracy with the aim of solving time-dependent partial differential equations in geophysical applications. As many problems suggest the use of staggered grids we investigate different schemes for the calculation of space derivatives on two separate grids. The differential operators are explicit and local in the sense that they use only information of the function in their nearest neighborhood, so that no matrix inversion is necessary. This makes this approach well-suited for parallelization. Differential weights are obtained either with the finite-volume method or using natural neighbor coordinates. Unstructured grids have advantages concerning the simulation of complex geometries and boundaries. Our results show that while in general triangular (hexagonal) grids perform worse than standard finite-difference approaches, the effects of grid irregularities on the accuracy of the space derivatives are comparably small for realistic grids. This suggests that such a finite-difference-like approach to unstructured grids may be an alternative to other irregular grid methods such as the finite-element technique.

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A HIGH-ORDER FAST MARCHING SCHEME FOR THE LINEARIZED EIKONAL EQUATION*

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We present a high-order upwind finite-difference scheme for solving a useful family of first-order partial differential equations, of which the linearized eikonal equation is a member. Fast solutions of the linearized eikonal equation have applications in travel-time tomography and residual migration algorithms. The technique, besides being both accurate and stable, escapes aperture limitations inherent in static marching schemes. We use a time-sequential evaluation method similar to Sethian's Fast Marching strategy to insure causal operator evaluation. We apply our technique to several complex slowness distributions, including the Marmousi model. We also use an adaptation of our technique to compute Cartesian-to-Ray coordinate transforms for the same slowness models.

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FREQUENCY DOMAIN WAVE PROPAGATION MODELING IN EXPLORATION SEISMOLOGY*

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To evaluate the wavefield for realistic 2-D and 3-D models we used a parallel computer, employing algorithms designed specifically to profit from the parallel architecture. The numerical procedures are iterative domain decomposition algorithms employing a non-conforming finite element, which are used to discretize the viscoacoustic and viscoelastic wave equations describing wave propagation in a porous medium saturated by either a single-phase or a two-phase compressible inviscid fluid and subject to absorbing boundary conditions at the artificial boundaries. Our purpose is to establish the effect of gas, brine or oil and gas-brine or gas-oil pore fluids on seismic velocities. Numerical examples showing the implementation of the algorithm to compute crosshole seismic response of simple 2-D and 3-D hydrocarbon reservoirs are presented.

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THREE-DIMENSIONAL TIME DOMAIN MODELING OF ULTRASONIC WAVE PROPAGATION IN CONCRETE IN EXPLICIT CONSIDERATION OF AGGREGATES AND POROSITY*

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Concrete as strongly heterogeneous and highly-packed composite material represents a very important but also very difficult object for ultrasonic nondestructive testing (NDT). Due to the high scatterer density, ultrasonic wave propagation in this material consists of a complex mixture of multiple scattering, mode conversion and diffusive energy transport. In order to obtain a better understanding of the effect of aggregates and porosity on elastic wave propagation in concrete and to optimize imaging techniques, e.g. synthetic aperture focusing technique (SAFT),¹ it is useful to model the wave propagation and scattering process explicitly in the time domain. In this paper, the three-dimensional EFIT-Code (EFIT: Elastodynamic Finite Integration Technique)² with periodic boundary conditions is used to model attenuation and dispersion of a plane longitudinal wave propagating in a synthetic three-dimensional concrete plate. Systematic parameter studies are carried out in order to demonstrate the effect of porosity and that of different aggregates. Finally, the simulation results are compared with former plane strain simulations, revealing significant differences in attenuation and signal-to-noise ratio between the two-dimensional and the more realistic three-dimensional case.

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TIME DOMAIN MODELING OF AXISYMMETRIC WAVE PROPAGATION IN ISOTROPIC ELASTIC MEDIA WITH CEFIT — CYLINDRICAL ELASTODYNAMIC FINITE INTEGRATION TECHNIQUE*

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The Elastodynamic Finite Integration Technique (EFIT), originally developed by Fellingner *et al.*,¹⁻³ represents a stable and efficient numerical code to model elastic wave propagation in linearly-elastic isotropic and anisotropic, homogeneous and heterogeneous as well as dissipative and nondissipative media. In previous works, the FIT discretization of the basic equations of linear elasticity, Hooke's law and Cauchy's equation of motion, was exclusively carried out in Cartesian coordinates. For problems in cylindrical geometries it is more suitable to use cylindrical coordinates. By that, axisymmetric problems can be treated in a two-dimensional staggered grid in the r, z -plane. The paper presents an EFIT version for axisymmetric problems in cylindrical coordinates called Cylindrical EFIT (CEFIT). After demonstrating the accuracy of the numerical code by a comparison between simulation results and analytical solutions, different examples of application are given. These examples include modeling of sound fields of ultrasonic transducers, thermoelastic laser sources, geophysical borehole probes, impact-echo measurements in layered media, and load simulations of the European Spallation Source (ESS) mercury target.

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ON THE PROPAGATION OF ACOUSTIC PULSES IN POROUS RIGID MEDIA: A TIME-DOMAIN APPROACH*

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Wave propagation of acoustic waves in porous media is considered. The medium is assumed to have a rigid frame, so that the propagation takes place in the air which fills the material. The Euler equation and the constitutive relation are generalized to take into account the dispersive nature of these media. We show that the connection between the fractional calculus and the behavior of materials with memory allows to work out time domain wave equations, the coefficients of which are no longer frequency dependent.

These equations are suited for direct and inverse scattering problems, and lead to the complete determination of the porous medium parameters.

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TRIANGLE-QUADRANGLE GRID METHOD FOR POROELASTIC, ELASTIC, AND ACOUSTIC WAVE EQUATIONS*

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A new numerical technique is developed for wave propagation in heterogeneous poroelastic media and mixed poroelastic, elastic and acoustic media. The scheme, based on a first-order hyperbolic Biot's system and a discretization mesh of triangles and quadrangles, solves the problem using integral equilibrium equations around each node, instead of satisfying Biot's differential equations at each node as in the finite-difference method. The surface topography and complex geometrical interfaces can be accurately modeled with the proposed algorithm by making the nodes of triangles and quadrangles follow the curved interfaces. The elastic (acoustic)/poroelastic interface conditions of complex geometry are introduced using the integral equilibrium equations around nodes at the interface based on the continuities of total stresses and velocities between the interface. The free-surface conditions of complex geometrical boundaries are satisfied naturally for the scheme. This work is an extension of the grid method for the heterogeneous elastic media to the heterogeneous poroelastic one. The proposed algorithm is successfully tested against an analytical solution for Lamb's problem when the algorithm is reduced to handle the elastic limit of the Biot's equations. Examples of wave propagation in a poroelastic half-space with a semi-cylindrical pit on the surface and mixed acoustic-poroelastic and elastic-poroelastic models with inclined interfaces are presented.

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ABSORBING LAYER VIA WAVE-EQUATION SPLITTING*

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Modeling acoustic waves generated by a localized source is always vexed by the nagging problem of spurious reflections and wraparound arising when the wavefront reaches the boundary of the numerical mesh. This difficulty may be circumvented by using a very large computational domain, which is very inefficient, or can be tackled by using some kind of absorbing boundary technique, which has not yet found a universally satisfactory solution. In this work, the wave equation is modified by introducing a term that is nonzero only in a narrow strip near the boundary. Then, a splitting technique permits to compute part of the solution analytically (hence, at no computational cost), while an application of Weyl's formula for the exponential of a matrix leads to a second-order accurate scheme that completes the algorithm. An application to SH seismic wave modeling shows that the performance of the present method is competitive with standard ones. Moreover, there is evidence for a potential application to the modeling of wave propagation in porous media, where stiff differential equations arise.

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OPTIMISED ABSORBING BOUNDARY CONDITIONS FOR ELASTIC-WAVE PROPAGATION*

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Second-order absorbing boundary conditions for numerical modeling of elastic-wave propagation are studied. The corresponding reflection coefficients are derived, from which a necessary and sufficient condition for complete absorption at normal incidence is deduced. We define a family of absorbing boundary conditions from symmetrically specified zero reflection incidences. Conditions to avoid singular reflection coefficients are given for this case, these ensure that the solutions of the elastic wave equation also satisfy the boundary conditions. These are then optimised over a wide range of materials, and absorbing boundary conditions that give an efficient absorption for the whole range are obtained. We also compare the results with absorbing boundary conditions developed from the least-squares solution of the system requiring complete absorption at all incidences. The best set of conditions are presented and compared with Clayton and Engquist⁶ (A2) condition.

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CONJUGATED INFINITE ELEMENTS FOR TWO-DIMENSIONAL TIME-HARMONIC ELASTODYNAMICS*

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Wave propagation problems in unbounded domains require the handling of appropriate radiation conditions (Sommerfeld). Various absorbing boundary conditions are available for that purpose. In a discrete finite element context, local and global Dirichlet-to-Neumann (DtN) and infinite element methods have shown their efficiency for the scalar wave equation.

The paper concentrates on the extension of an infinite element method to the elastodynamic vector wave equation. The extension is developed in the frequency domain for 2-D problems. The paper focuses on the development of a conjugated formulation using the Helmholtz decomposition theorem of smooth vector fields. The accuracy of the developed formulation is assessed through the study of benchmarks. The computed results are shown to be in good agreement with the analytical solution for a multi-pole field along a circular cavity and with the results produced by other numerical methods.

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