USING GAUSSIAN BEAM MODEL IN OCEANS WITH PENETRATING SLOPE BOTTOMS

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A numerical code using Gaussian Beam Model (NTUGBM) is developed for underwater acoustic propagation at high frequency (larger than 1 kHz) in oceans with penetrating slope bottom. Several test cases are used to benchmark NTUGBM. Cases include continental shelf and continental slope. The results of NTUGBM are compared with results using EFEPE and FOR3D (Nx2D version). Results of NTUGBM agree well with those of both codes.

1 Introduction

In order to accurately and efficiently simulate the acoustic field, some sorts of numerically methods have been developed. In this paper, a numerical model called NTURAY, which is developed using the Gaussian Beams Method, is illustrated [1]. The propagation models deduced from the Helmholtz Equation are classified in Fig. 1, which are divided into the range-dependent and the range-independent models. Our goal is to establish a high-frequency, range-dependent numerical model with the capability to accomplish the long-range ray tracing and transmission loss calculations in the laterally varying multi-layered ocean environment. According to the requirements, only the Ray method is efficient enough to handle the high-frequency and ray tracing computation.

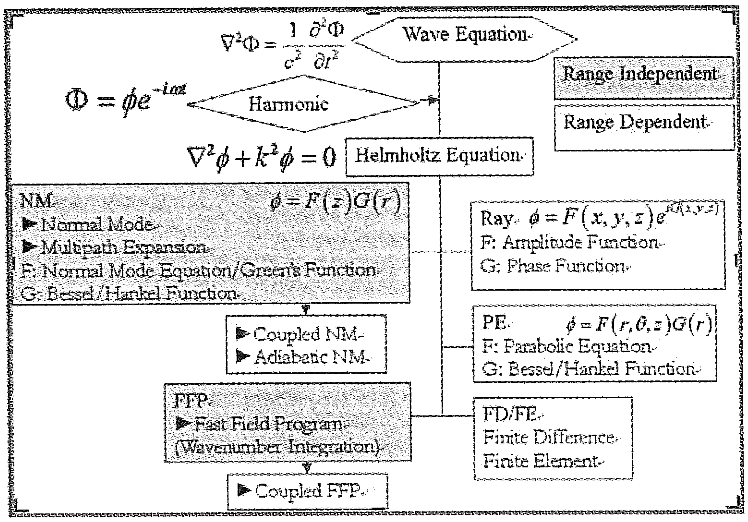

Figure 1. The propagation models deduced from the Helmholtz Equation.

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A serious drawback is using the Ray Method in the vicinity of caustics, and Gaussian Beams Method can overcome this problem and effectively calculate the transmission loss in caustics and shadow zone as well. Cerveny et al. [2-4] first applied the Gaussian Beams Method in geophysics, and then this method is used in underwater acoustics application by Porter and Bucker [5] and Weinberg and Keenan [6]. All the applications introduced above dealt with a flat bottom, so the contribution of this paper is to apply the Gaussian Beans Method in cases of slope bottom and laterally varying layered bottom.

In section 2, we will introduce the theory of Gaussian Beams Method. The verification of the NTURAY model is discussed in section 3, and section 4 will talk about the calculation of the layered bottom. Finally, section 5 will give brief discussions and conclusions.

2 Gaussian Beam Method

The linear acoustic wave equation is written as

$$\nabla^2 P = \frac{1}{c^2}\frac{\partial^2 P}{\partial t^2} \tag{1}$$

For a harmonic wave, the solution to the linear acoustic wave equation is

$$P(\vec{x},t) = A(\vec{x})e^{i\omega[t-\tau(\vec{x})]}, \tag{2}$$

where $\varpi(t-\tau(\vec{x}))$ is the constant phase surface, ϖ is the frequency and $A(\vec{x})$ is the amplitude. Substitute Eq. (2) into Eq. (1), we can obtain the following equation,

$$\left\{(\nabla^2 A - \omega^2 A|\nabla\tau|^2 + \frac{\omega^2}{c^2}A) + i\left(2\omega\nabla A\cdot\nabla\tau + \omega A\nabla^2\tau\right)\right\}e^{i\omega(t-\tau)} = 0 \tag{3}$$

The real part and imaginary part are equal to zero as following,

$$\frac{\nabla^2 A}{\omega^2} - A|\nabla\tau|^2 + \frac{A}{c^2} = 0 \tag{4}$$

$$2\nabla A\cdot\nabla\tau + A\nabla^2\tau = 0 \tag{5}$$

Thus, if the amplitude changes slightly with the space and if the frequency is high enough, Eq. (4) becomes Eq. (6)

$$|\nabla\tau|^2 = \frac{1}{c^2} \tag{6}$$

Because the directional vector of the ray is

$$\widehat{e}_t = \frac{d\vec{x}}{ds} = \frac{\nabla\tau}{|\nabla\tau|}, \tag{7}$$

combining this directional vector with Eq. (6), the Eikonal Equation can be deduced to be Eq. (8).

$$\frac{d}{ds}\left(\frac{1}{c}\frac{d\bar{x}}{ds}\right)=-\frac{1}{c^2}\nabla c \tag{8}$$

Thus we can obtain the geometry of acoustic rays by solving the functions in cylindrical coordinate (Nx2D calculation, eliminate the θ coupling)

$$\begin{cases}\dfrac{d}{ds}\left(\dfrac{1}{c}\dfrac{dr}{ds}\right)=-\dfrac{1}{c^2}\dfrac{\partial c}{\partial r}\\[2ex]\dfrac{d}{ds}\left(\dfrac{1}{c}\dfrac{dz}{ds}\right)=-\dfrac{1}{c^2}\dfrac{\partial c}{\partial z}\end{cases} \tag{9}$$

Giving that $\frac{1}{c}\frac{dr}{ds}=\xi$ and $\frac{1}{c}\frac{dz}{ds}=\zeta$, Eq. (9) becomes

$$\begin{cases}\dfrac{dr(s)}{ds}=c(s)\cdot\xi(s)\\[2ex]\dfrac{dz(s)}{ds}=c(s)\cdot\zeta(s)\\[2ex]\dfrac{d\xi(s)}{ds}=-\dfrac{1}{c^2(s)}\dfrac{\partial c(s)}{\partial r}\\[2ex]\dfrac{d\zeta(s)}{ds}=-\dfrac{1}{c^2(s)}\dfrac{\partial c(s)}{\partial z}\end{cases} \tag{10}$$

Thus we can solve the equation system simultaneously with the initial conditions to obtain the ray traces.

If we rewrite the solution of the standard linear wave equation as

$$P(\bar{x},t)=A(\bar{x})e^{i\varpi[t-\tau(\bar{x})]}=u(\bar{x})e^{i\varpi t}, \tag{11}$$

then the solution in ray-centered coordinate system can be represented as

$$u(s,n)=A_0\cdot\sqrt{\frac{c(s,0)}{q(s)r}}\cdot e^{\frac{-n^2}{L^2(s)}}\exp\left[-i\omega\left(\tau(s)+\frac{K(s)}{2c(s,0)}n^2\right)\right] \tag{12}$$

where

$$L=\left[-\frac{\omega}{2}\mathrm{Im}\left(\frac{p(s)}{q(s)}\right)\right]^{\frac{-1}{2}},\ K=c(s,0)\cdot\mathrm{Re}\left(\frac{p(s)}{q(s)}\right)$$

Therefore, we will obtain the sound field, $u(\bar{x})$, by calculating two system parameters $p(s)$ and $q(s)$,

$$\begin{cases} q_{,s} = \hat{c}p \\ p_{,s} = -\dfrac{c_{,nn}\big|_{n=0}}{\hat{c}^2} q \end{cases} \tag{13}$$

and the transmission loss is

$$TL = 20 \log |u| \tag{14}$$

3 Verification of the NUTRAY Model

We verify the accuracy of the NTUTAY model by comparing the solutions of the NTURAY model and the analytic solution [6] and that of the EFEPE model. Case 1 is a free space case, the NTURAY solution is compared with the analytical solution and the result shows that they match very well, as shown in Fig. 2. Similar to case 1, case 2 and case 3 are the comparisons between the NTURAY solutions and the analytical solutions of the half-space case and the shallow water, hard-bottom waveguide case (See Fig. 3 and Fig. 4). These cases show that the NTURAY model is accurate and acceptable in the simple, basic condition.

Case 4 ~6 are the comparisons between the NTURAY solutions and the EFEPE solutions. Case 4 is a shallow water waveguide case with penetrable bottom (as shown in Fig. 5), and both case 5 and case 6 are cases of continental shelf (slope of 1/500, see Fig. 6) and continental slope (slope of 1/20, see Fig. 7) , respectively. Fig. 8 ~10 represent the results of case 4 ~6.

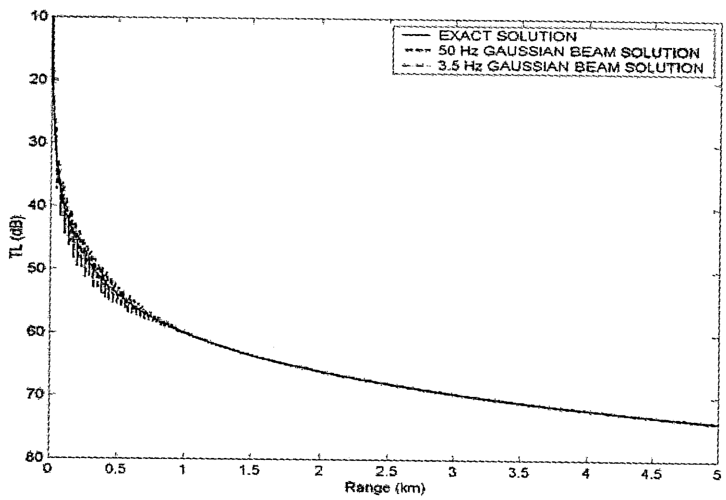

Figure 2. The results of free space case. The NTURAY solution is compared with the analytical solution.

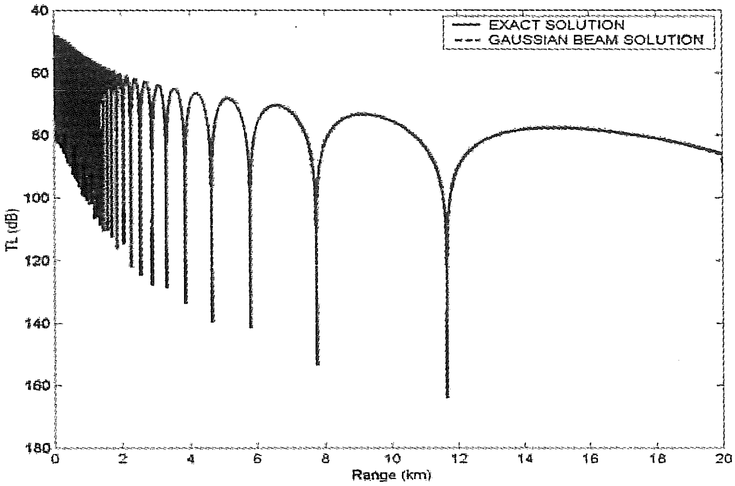

Figure 3. The results of half space case. The NTURAY solution is compared with the analytical solution.

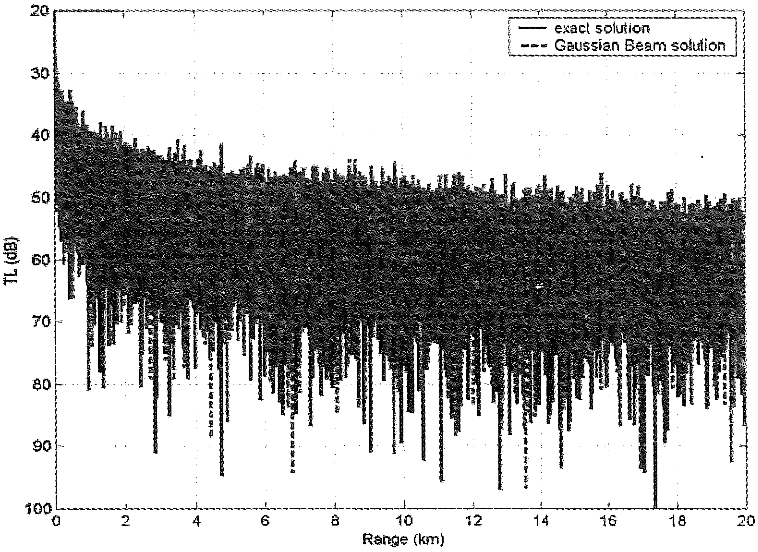

Figure 4. The results of the shallow water, hard-bottom waveguide. The NTURAY solution is compared with the analytical solution (the normal mode solution).

All cases show that the NTURAY solutions match well with others except case 6. Two solutions in case 6 begin to deviate from 5 km and further away from the source. This may be due to the EFEPE model is not developed to calculate high-frequency sound field. Compare the EFEPE solution with another PE model, the FOR3D model [8], the discrepancy still exists between two PE models. Thus the NTURAY model still needs to be compared with the other well-developed, high-frequency model.

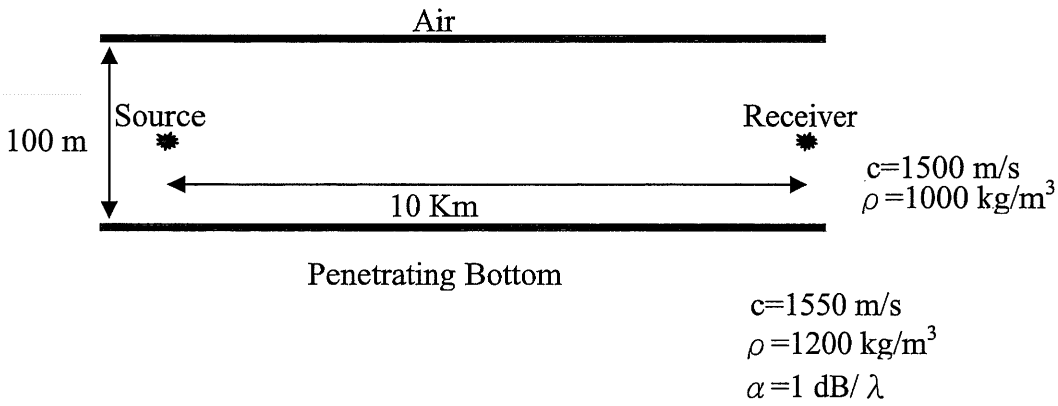

Figure 5. Case 4: the shallow water waveguide with a penetrable bottom.

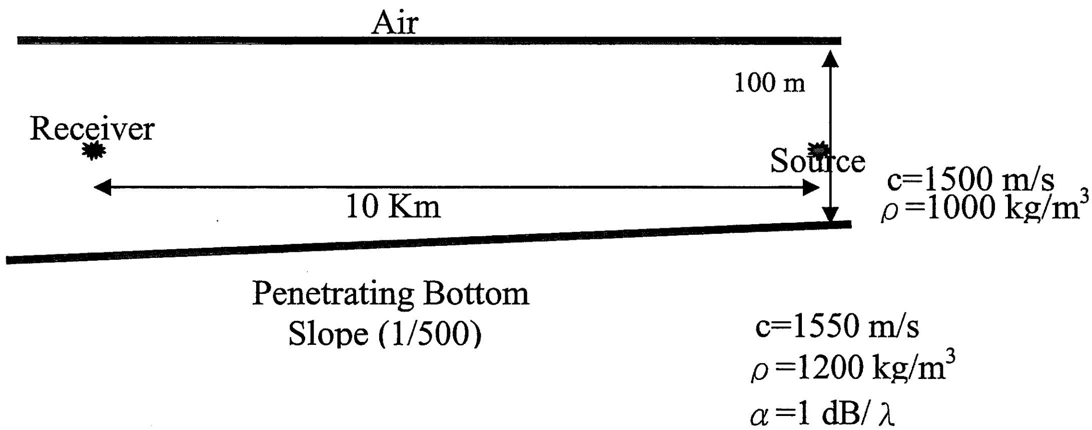

Figure 6. Case 5: the case of continental shelf (slope is equal to 1/500) with penetrable bottom.

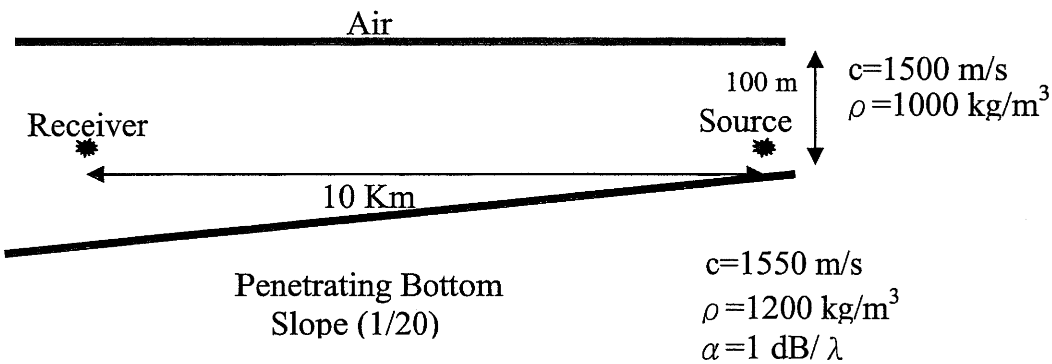

Figure 7. Case 6: the case of continental slope (slope is equal to 1/20) with penetrable bottom.

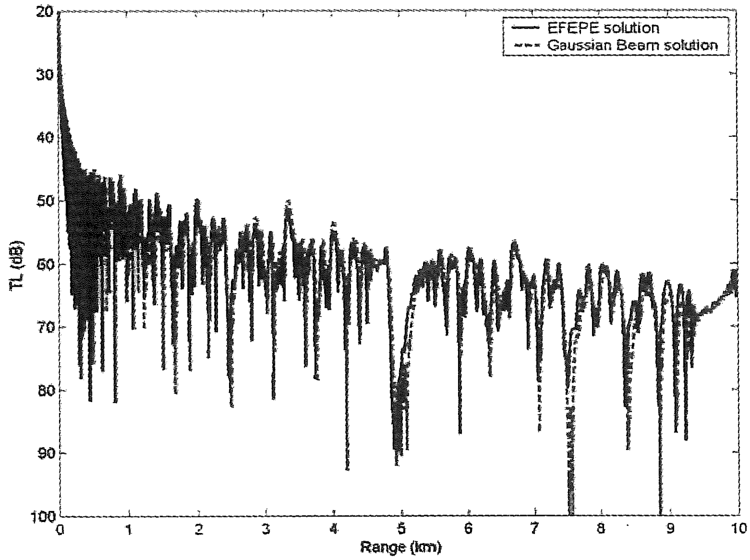

Figure 8. The results of shallow water waveguide with penetrable bottom. The NTURAY solution is compared with the EFEPE solution, and they math quite well.

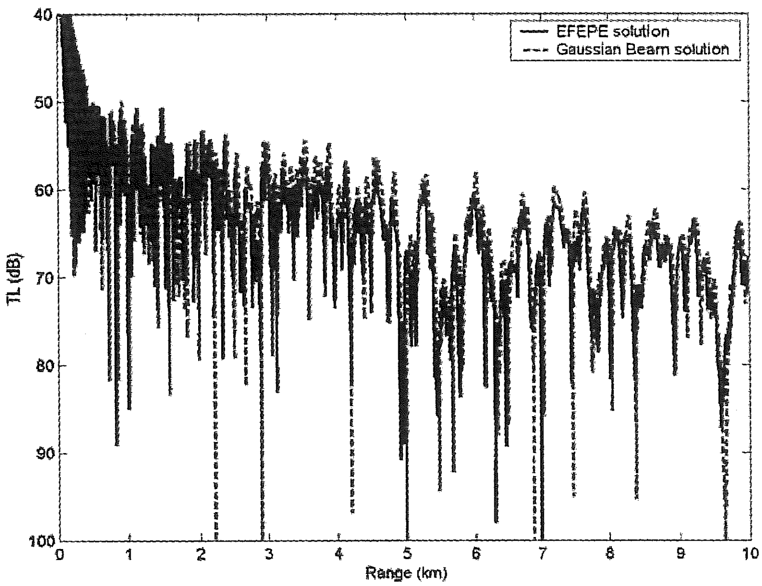

Figure 9. The results of continental shelf case (slope is equal to 1/500) with penetrable bottom. The NTURAY solution is compared with the EFEPE solution and match very well.

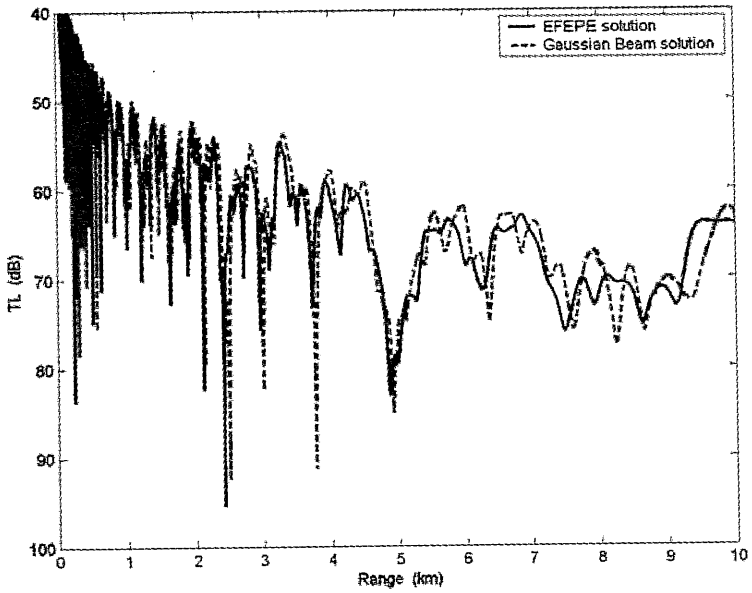

Figure 10. The results of the continental slope case (slope is equal to 1/20) with penetrable bottom. The NTURAY solution is compared with the EFEPE solution; two solutions start to diverge from about 5 km far from the source.

4 Two-layered Bottom Model

According to Frisk [7], the summarized reflection effect of the two layer bottom (as shown in Fig. 11) can be represented by the Rayleigh Reflection Coefficient

$$R = \frac{R_{01} + R_{12} e^{2ik_1 h \cos \theta_1}}{1 + R_{01} R_{12} e^{2ik_1 h \cos \theta}} \tag{15}$$

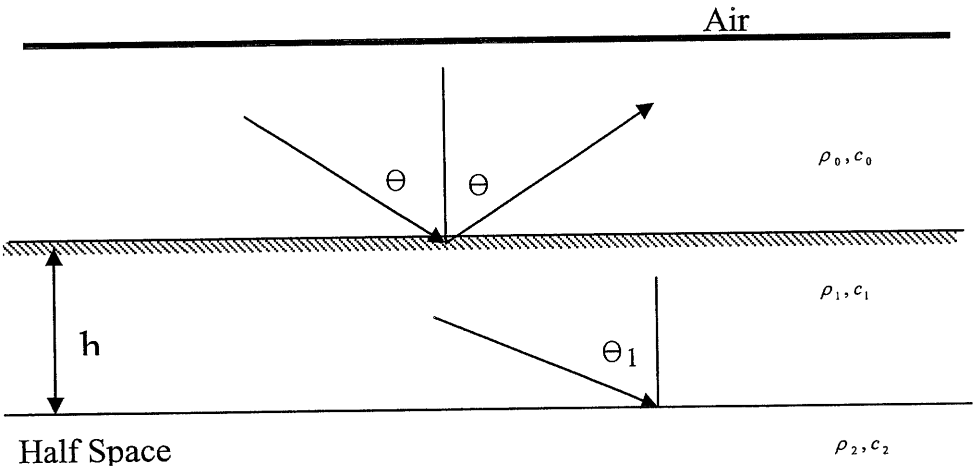

Figure 11. The two layer bottom.

The results of the two-layered NTURAY model and the EFEPE model are presented in Fig. 12. It shows that the two-layered NTURAY model is accurate in this case.

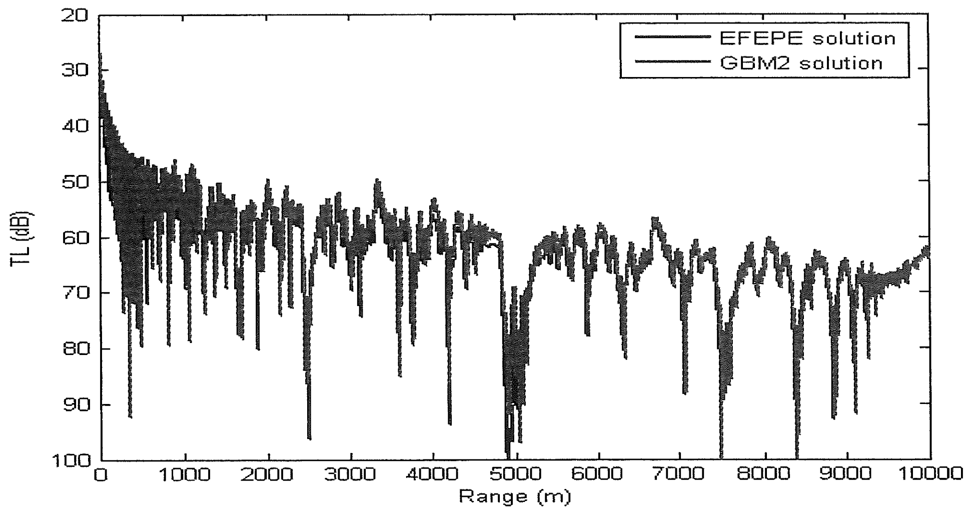

Figure 12. The results of the two-layered bottom case. The NTURAY solution is compared with the EFEPE solution.

5 Conclusion

In this paper, the NTURAY model is proposed. The NTURAY is a range-dependent model which uses the Gaussian Beams Method to calculation the high-frequency, long range acoustic field. It can deal with the laterally varying multi-layered ocean environment and calculate the traces and the transmission loss.

Several cases are used to verify the accuracy of the NTURAY, and the results show that the comparisons between analytic solutions or EFEPE solutions and NTURAY solutions are satisfactory.

6 Reference

1. C. F. Chen, W. S. Hwang, L. W. Hsieh, and Y. T. Lin, "Verification and evaluation of the advanced sonar range prediction system (ASORPS)," National Taiwan University, report of Engineering Science and Ocean Engineering, Underwater Acoustic Laboratory, UAL-NTU TR 0101, 2002.
2. V. Cerveny, M. M. Popov and I. Psencik, "Computation of wave fields in inhomogeneous media – Gaussian beam approach," Geophys. J. astr. Soc. 70, pp.109-128, 1982.
3. V. Cerveny and I. Psencik, "Gaussian beams in two–dimensional elastic inhomogeneous media," Geophys. J. astr. Soc. 72, pp. 417-433, 1983.
4. V. Cerveny and I. Psencik, "Gaussian beams in 2–D laterally varying layered structures," Geophys. J. astr. Soc. 78, pp. 65-91, 1984.

5. M. B. Porter and H. P. Bucker, "Gaussian beam tracing for computing ocean acoustic fields," J. Acoust. Soc. Am. 82(4), pp.1349-1359, 1987.
6. H. Weinberg and R. Keenan, "Gaussian ray bundles for modeling high-frequency propagation loss under shallow water conditions," J. Acoust. Soc. Am. 100(3), 1996.
7. G. V. Frisk, Ocean and Seabed Acoustics: a theory of wave propagation, Prentice-Hall, NJ, 1994.
8. D. Lee and M. H. Schultz, "*Numerical Ocean Acoustic Propagation in Three Dimensions,*" Singapore: World Scientific, pp. 138-144, 1995.