RECONSTRUCTION OF SEISMIC IMPEDANCE FROM MARINE SEISMIC DATA

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In this paper we focus our attention on the Marchenko inversion method which requires as input the reflectivity sequence of the medium with the view to reconstructing the seismic impedance from seismic reflection data. The reflectivity sequence and the relevant seismic wavelet are extracted from marine reflection data by applying the statistical estimation procedure known as Markov Chain Monte Carlo method to the problem of blind deconvolution. In order to implement the inversion method, the assumption of pure spike trains that was used previously has been replaced by amplitudes having a narrow bell-shaped form to facilitate the numerical solution of the Marchenko integral equation from which the underlying profile of the medium is obtained. Various aspects of our inversion procedure are discussed. These include questions related to the handling of experimental data and the numerical solution of the Marchenko integral equation using piecewise polynomials.

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I. INTRODUCTION

Various methods for seismic exploration have been employed in the past to extract information on subsurface properties of the Earth. The most commonly applied is the seismic reflection method in which both the source and receiver are spread out on the surface. The success of this method is mainly due to the multi-layered structures of sedimentary basins, which reflect the seismic wavelet back to the surface. In this work we will consider marine exploration only.

In order to obtain quantitative information on subsurface properties, in particular, the seismic impedance, we employ the Marchenko integral equation (MIE) [1–3]. The method is closely connected to the inverse problem [4, 5] and its historical evolution can be found, for example, in Refs. [2, 6, 7]. A complete bibliography of pioneering papers dealing with the inverse problem can also be found in Faddeev's paper [5] and in references therein. As indicated by its name, the inverse scattering problem has a counterpart known as the direct scattering problem, in which one proceeds from the potential to the scattering data. Thus the methodology used for solving the inverse problem relies strongly on the formulation of the direct problem.

For most practical situations in the seismic reflection method, the Earth can be considered as an elastic medium. The elastic wave equation which can be transformed into a Schrödinger-type equation is therefore adequate for the direct problem. This in turn allows treatment via the Marchenko inverse scattering method.

The solution of the MIE requires as input the reflectivity sequence of the medium which can be extracted from the marine reflection data. This can be achieved by applying the Markov Chain Monte Carlo (MCMC) method [8–11] based on the Gibbs sampler to iteratively generate random samples from the joint posterior distribution of the unknowns. The MCMC method is based on Bayesian analysis and provides a general mechanism to sample the parameter vector from its posterior distribution via the Monte Carlo method.

In section II the blind deconvolution approach which uses the MCMC method as an alternative form for simultaneously deconvolving the seismic wavelet and reflectivity sequence from marine reflection data is discussed. In section III the inverse reflection problem and the Marchenko inversion method are briefly described. Calculations and results are given in section IV while the conclusions are summarized in section V.

II. BLIND DECONVOLUTION

A. Deconvolution process

Before discussing the deconvolution process we present a brief description of the convolution model. This model can be described schematically as [12]

$$\text{measured output} = \text{output} + \text{noise} = \text{wavelet} * x + \text{noise},$$

where x is the reflectivity sequence. Mathematically, it can be written as

$$z_t = \sum_{k=1}^{\min(N,t)} h_k x_{t-k+1} + n_t\,, \qquad t = 1, ..., N+M-1\,, \tag{1}$$

where z is an observed seismic trace of length $N+M-1$, h represents the seismic wavelet of length N, x stands for the white reflectivity sequence of the medium of length M and n is a zero-mean white noise of Gaussian type. The noise sequence is characterized by its variance σ^2 [12]. Eq. (1) can be written in a convolutional form

$$z = h * x + n\,. \tag{2}$$

Our objective is to seperate the reflectivity sequence and seismic wavelet from each other by applying the blind deconvolution procedure.

In the literature the system's unit response is called the reflectivity sequence. In our model it will also include multiple reflections (only a finite number is needed) effected by the system provided the seismic wavelet is shorter than the travel time distance between the consecutive interfaces. For our numerical computations we identify the reflectivity sequence up to a scaling factor with the unit response of the medium $B(\xi)$, which is discussed in section III.

Deconvolution of the seismic reflection data series z when the source wavelet h is known, is a well-understood problem; however, in some investigations such as ours, only the marine seismic reflection data have been provided and both reflectivity and seismic wavelet should be retrieved from them. In order to estimate these quantities, we apply the blind deconvolution method.

We assume the sea floor to consist of several homogeneous layers that are separated by interfaces. Such an assumption makes it possible to express the reflectivity sequence in terms of a Bernoulli-Gaussian (BG) sequence [13, 14]. Thus, the reflectivity sequence that defines the generalized BG sequence can be modeled by using two random sequences expressed as [12]

$$x_k = r_k q_k \,, \tag{3}$$

where $r = (r_k)$ denotes a zero-mean Gaussian white sequence with variance σ_r^2 and $q = (q_k)$ stands for the Bernoulli sequence with the probability parameter λ being equal to its mean value [15]. For the probabilities associated with this sequence we have

$$P(q_k = 1) = \lambda \,, \tag{4}$$

or

$$P(q_k = 0) = 1 - \lambda \,, \tag{5}$$

that is, the random variable q_k is one with probability λ and zero with probability $1 - \lambda$. The probability of the whole sequence q reads

$$P(q|\lambda) = \prod_k P(q_k) = \lambda^n (1-\lambda)^{M-n}, \tag{6}$$

where n is the number of ones in the sequence.

B. Markov Chain Monte Carlo method

We are concerned with the MCMC method in a pure Bayesian approach. Upon using the Bayes' rule, we can write the probability distribution in the form [16]

$$P(\theta|z) = \frac{P(z|\theta)P(\theta)}{P(z)} \,, \tag{7}$$

where θ stands for all parameters of the problem [17], that is,

$$\theta \equiv (h, x, \lambda, \sigma^2) \,, \tag{8}$$

and h, x, λ, and σ^2 have the same meaning as above. $P(\theta|z)$ is the posterior probability of the model conditional on the observed data z, $P(\theta)$, and $P(z)$ describe the prior knowledge and seismic reflection data respectively while $P(z|\theta)$ describes the discrepancy between the model and observation.

The complete joint probability distribution is expressed in the form [16]

$$P(\theta|z) \propto P(z|\theta)P(\theta) \,, \tag{9}$$

since $P(z)$ is in this case a normalizing constant. The MCMC algorithm is iterative and may require a number of iterations before the first sample from the correct distribution can be provided. These initial iterations are called burn-in iterations and should not be used in the statistical analysis. Thus, the estimation of reflectivity sequence and the seismic wavelet is determined by the simulation of random variables via the MCMC algorithms based on the Gibbs sampler [9], which is regarded as the best known and most popular of the MCMC algorithms [10]. It is an algorithm in which the vector $\theta^{(k+1)}$ is obtained from $\theta^{(k)}$ by updating the vector elements one at a time. The prior distribution in Eq. (9) can be written as

$$P(\theta) = P(x|\lambda)P(\lambda)P(h)P(\sigma^2)\,, \tag{10}$$

where $P(h) = P(\sigma^2) = P(\lambda) = 1$ are flat prior probability densities and $P(x|\lambda)$ is described by a Bernoulli-Gaussian distribution. Upon using Eq. (10), we can write Eq. (7) in the form

$$P(\theta|z) = \frac{P(z|\sigma^2, x, \lambda)P(x|\lambda)P(h)}{P(z)}\,. \tag{11}$$

The assumption of a white Gaussian noise sequence n of variance σ^2 leads to

$$P(z|\sigma^2, x, \lambda) = (2\pi\sigma^2)^{-(N+M-1)/2} \exp\left(-\frac{\|z - h * x\|^2}{2\sigma^2}\right)\,. \tag{12}$$

For our purpose, we need to calculate the distributions $P(h|z, x, \sigma^2, \lambda)$, $P(x|z, h, \lambda, \sigma^2)$, $P(\sigma^2|z, x, h, \lambda)$, and $P(\lambda|z, x, h, \sigma^2)$. Thus, Eq. (11) can be employed to handle the relevant re-sampling processes.

1. Re-sampling of the amplitude of the reflectivity sequence

The reflectivity sequence x contains information about the Earth's structure. In order to statistically separate it from the seismic wavelet we use the BG white sequence model [13]: $P(x) = \prod_m P(x_m)$, with

$$P(x_m) \sim (1 - \lambda)\delta(x_m = 0) + \lambda\mathcal{N}(0, \varrho^2)\,, \tag{13}$$

where $\mathcal{N}$ is a Gaussian distribution with specified mean and variance and where $\lambda \in (0, 1)$ is the probability that $x_m = 1$, and both λ and ϱ^2 are unknown.
It can be shown [13] that the posterior probabilities involving single components of the vector x remain Gaussian mixtures with a structure comparable to that of Eq. (13). More precisely,

$$P(x_m|\lambda, h, z, x_{-m},) \sim (1 - \lambda_m)\delta(x_m = 0) + \lambda_m\mathcal{N}(x_m^*, \sigma_m^2)\,, \tag{14}$$

where $x_{-m} = (x_1, \ldots, x_{m-1}, x_{m+1}, \ldots, x_M)$ and

$$\sigma_m^2 = \frac{\sigma^2\varrho^2}{\sigma^2 + \varrho^2\|h\|^2}\,, \tag{15}$$

$$x_m^* = \frac{\sigma_m^2}{\sigma^2}(h^T e_m)\,, \tag{16}$$

$$\lambda_m = \left[1 + \frac{\lambda}{1-\lambda}\frac{\sigma_m}{\varrho}\exp\left(\frac{x_m^{*\,2}}{2\sigma_m^2}\right)\right]^{-1}\,, \tag{17}$$

and

$$e_m = z - h * x^{(m)}\,, \tag{18}$$

where $x^{(m)}$ is identical to x except for $x_m^{(m)} = 0$. Using Eqs. (14)-(17), the components x_m of the reflectivity sequence can be re-sampled, one at a time.

2. Re-sampling of the seismic wavelet

In order to re-sample the seismic wavelet h, we deduce from the Bayes rule that $P(h|\sigma^2, x, z) \propto P(z|\sigma^2, x, h)$, given $P(h) = 1$, where $P(z|\sigma^2, x, h)$ is given by Eq. (12). Moreover, it is easy to check the following identity:

$$-\frac{\|z - h * x\|^2}{2\sigma^2} = -\frac{1}{2}(h-\mu)^T R^{-1}(h-\mu), \tag{19}$$

where

$$\mu = (X^T X)^{-1} X^T z\,, \tag{20}$$

and

$$R = (X^T X)^{-1}\sigma^2 1\,, \tag{21}$$

where X is the Toeplitz matrix of size $(N+M-1, N)$ such that

$$Xh = h * x\,. \tag{22}$$

This allows us to conclude that the posterior probability of h is a multivariate Gaussian with mean vector μ and with covariance matrix R. The latter probability is easy to sample according to $h = \mu + Q\epsilon$, where ϵ is a normalized Gaussian white noise and Q^T is a square root matrix of R (that is, such that $R = QQ^T$), such as the one resulting from the Cholesky decomposition.

3. Re-sampling of the hyperparameter σ

Given $P(\sigma^2) = 1$, it is also true that $P(\sigma^2|z, x, h, \lambda) \propto P(z|\sigma^2, x, h)$. As a function of σ^2, Eq. (12) takes the form

$$P(z|\sigma^2, x, \lambda) = \frac{1}{(\sigma^2)^{\alpha+1}} \exp(-\beta/\sigma^2)$$

up to a multiplicative constant, with $\alpha = (N+M-1)/2-1$ and $\beta = \|z - h * x\|^2/2$, which means that the posterior probability of σ^2 follows an inverse gamma distribution of parameters (α, β). The latter can be easily sampled by taking the inverse of a gamma random generator output with the same parameters.

4. Re-sampling of the hyperparameter λ

The reflectivity sequence x gathers all the information about λ contained in (z, x, h, σ^2), that is, $P(\lambda|z, x, h, \sigma^2) = P(\lambda|x)$. Following [13], let us remark that the Bernoulli sequence q can be retrieved from x with probability one according to $q_k = 1$ if $x_k \neq 0$, $q_k = 0$ otherwise. Thus, $P(\lambda|x) = P(\lambda|q)$, the latter being proportional to $P(q|\lambda)$ since we assumed a flat prior $P(\lambda) = 1$. Finally, according to Eq. (6), we get

$$P(\lambda|z, x, h, \sigma^2) \propto \lambda^n (1-\lambda)^{M-n}\,, \tag{23}$$

which belongs to the family of beta probability densities $\mathcal{B}(\alpha, \beta)$ with $\alpha = n+1$ and $\beta = M-n+1$.

III. INVERSE REFLECTION PROBLEM

The one-dimensional seismic wave equation for the elastic displacement u is given by [18]

$$\rho\frac{\partial^2 u}{\partial t^2} - \frac{\partial}{\partial z}\left(\rho c^2 \frac{\partial u}{\partial z}\right) = 0\,, \tag{24}$$

where t is the time, z is the space coordinate along the direction of propagation, $\rho = \rho(z)$ is the density of the medium, and $c = c(z)$ is the speed of the seismic wave. We are considering here a longitudinal displacement in the z-direction. The Marchenko integral equation is directly applicable to the case of inversion with a seismic wave normally incident on a planar stratified medium, provided that the one-dimensional seismic wave equation is converted to the Schrödinger equation. Thus, the coordinate variable z is changed to the travel time ξ defined by

$$\frac{\partial \xi}{\partial z} = \frac{1}{c(z)}\,. \tag{25}$$

When integrating Eq. (25) we obtain

$$\xi = \int_0^z \frac{1}{c(z')}dz'\,, \tag{26}$$

which is the travel time for a pulse to move from the origin to position z. Upon using this relation we can rewrite the wave equation as

$$\eta(\xi)\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial \xi}\left(\eta(\xi)\frac{\partial u}{\partial \xi}\right)\,, \tag{27}$$

where $\eta(\xi) = \rho c$ is the seismic impedance of the medium. Defining ψ via $\psi = \sqrt{\eta}u$ we obtain

$$\frac{\partial^2 \psi}{\partial \xi^2} - \frac{\partial^2 \psi}{\partial t^2} = V\psi\,, \tag{28}$$

where V is given by

$$V(\xi) = \frac{1}{\sqrt{\eta}}\frac{d^2\sqrt{\eta}}{d\xi^2}\,. \tag{29}$$

For an ansatz of the form $\psi(\xi,t) = \exp(-ikt)f(\xi)$ the Schrödinger-type equation

$$\frac{d^2 f}{d\xi^2} + (k^2 - V(\xi))f = 0\,, \tag{30}$$

is obtained. From the definition of V in Eq. (29) we can write

$$\frac{d^2\sqrt{\eta}}{d\xi^2} - V(\xi)\sqrt{\eta} = 0\,, \tag{31}$$

which is a reduced form of Eq. (30) with $k = 0$. For the inversion procedure we apply the Marchenko integral equation given as [2]

$$K(\xi,t) + B(\xi+t) + \int_{-\xi}^{+\xi} dt' K(\xi,t')B(t+t') = 0, \qquad |t| \leq \xi\,, \tag{32}$$

for which $K(\xi,t) = 0$, for $|t| > \xi$, and it denotes a non-causal function while the function B is causal and represents a reflectivity sequence. The function K satisfies the wave equation given by Eq. (28). Thus,

$$K(\xi,-\xi) = 0\,, \tag{33}$$

and

$$V(\xi) = 2\frac{dK(\xi,\xi)}{d\xi}\,. \tag{34}$$

The output kernel $K(\xi,\xi')$ can be determined by using the collocation method and piecewise polynomials, in our case Hermite splines. The Schrödinger-type equation, Eq. (30), is equivalent to the Marchenko integral equation via Eq. (34). The seismic impedance η can be calculated from the potential $V(\xi)$ in Eq. (29) or directly from the relation [19]

$$\eta(\xi) = \eta(0)\left(1 + \int_{-\xi}^{+\xi} K(\xi,\xi')\mathrm{d}\xi'\right)^2\,. \tag{35}$$

This means that, given the $\eta(0)$, the seismic impedance $\eta(\xi)$ for $\xi > 0$ can be recovered from the knowledge of the kernel $K(\xi,t)$.

IV. RESULTS

We illustrate the use of the blind deconvolution method on recorded marine seismic reflection data derived from a seismic survey in a deep water location in the North sea. We use data collected with a streamer containing 240 hydrophone groups. The group interval is 15 m. The sampling rate is 4 ms and the total length is 8 s. Each trace is composed of 2001 samples as shown in Figs. 1 and 2. Depicted in Fig. 3 are the seismic reflection data. We migrated the seismic reflection data using the standard moveout correction method [20] (or any other standard textbook) with the result as shown in Fig. 4.

The main modification as compared to Chen's version [13] is that we assume a shape given by

$$s = [0.1,\ 0.4,\ 1.0,\ 0.4,\ 0.1]\,, \tag{36}$$

instead of pure spikes in order to have a narrow bell-shaped form to facilitate the numerical solution of the Marchenko inversion. This means that the observation model, Eq. (2), now becomes

$$z = h * s * x + n\,, \tag{37}$$

where s is a known shape. If the shape s is equal to unity then of course the original equation (2) is recovered. We use in our calculations the seismic dataset from Fig. 4 which only include seismic traces from groups 90 to 240 since the normal moveout correction method did not give satisfactory results for the other groups because their offsets are too large. In addition we used all migrated traces collectively as observed seismic reflection data and modified the MCMC algorithm accordingly.

We also note, that the observed seismic reflection data are not calibrated, that is, they only provide relative amplitudes. The details of the source of the signal, that is, the airgun, are also not known. Therefore, the reflectivity sequence that we obtain from the statistical procedure will be related to the unit response of the medium by a suitable factor. This calibration problem can be solved by using additional information, such as the seismic impedance jump at the ocean bottom if known via other means.

For our purpose we model the sea floor as fluid so that only the compressional seismic wave can be supported. If we assume that the sea bottom consists of silt (fine sand or soil) and that the density changes much more than the velocity, then we can write [21–23]

$$\frac{\rho_2}{\rho_1} = 1.7\,, \tag{38}$$

where $\rho_1 = 1000\,\mathrm{kg\,m^{-3}}$ is the density of the sea water and $\rho_2 = 1700\,\mathrm{kg\,m^{-3}}$ is the density of silt. Similarly, if we assume that the velocity of sound does not change much, then we obtain the ratio

$$\frac{c_2}{c_1} = 1.05\,, \tag{39}$$

where $c_1 = 1500\,\mathrm{m\,s^{-1}}$ is the velocity of the seismic compressional wave in sea water and $c_2 = 1575\,\mathrm{m\,s^{-1}}$ is the velocity of the seismic compressional wave in silt. Thus, the seismic impedance is expressed in the form

$$\frac{\eta_2}{\eta_1} = \frac{\rho_2 c_2}{\rho_1 c_1} = 1.785\,, \tag{40}$$

where η_1 is the seismic impedance of the sea water and η_2 is the seismic impedance of silt. Assuming this value of the ratio η_2/η_1 we proceed to re-scale the amplitude of the estimated reflectivity sequence by a suitable factor. This factor is obtained by scaling the amplitude of the input kernel into the Marchenko equation, such that the inversion procedure yields a first jump approximately equal to $\eta_2/\eta_1 = 1.785$.

Shown in Fig. 5 is the seismic wavelet extracted from the migrated seismic reflection data in Fig. 4. Fig. 6 depicts the statistically retrieved reflection sequence corresponding to the seismic wavelet in Fig. 5. The estimated seismic impedance is shown in Fig. 7. Fig. 8 shows the reflectivity sequence scaled by a suitable factor and the corresponding estimated seismic impedance is shown in Fig. 9, while in Fig. 10 we observe the invariance of peak ranking and location of peaks between the estimated seismic impedances with and without a scaling factor. Thus a lot of information can be retrieved even without knowledge of the proper scaling factor.

V. CONCLUSIONS

We have presented the blind deconvolution of the Marine seismic reflection data wherein the Bernoulli-Gaussian white sequence model for the reflectivity sequence has been used. We presented an MCMC method for simultaneously estimating seismic wavelet and reflectivity sequence under the Bayesian approach. With the estimated reflectivity sequence at hand, the seismic impedance of the Earth medium has been reliably estimated by applying the Marchenko inverse scattering method.

However, since the marine seismic reflection data are not calibrated and the details of the source signal are not known, we related the acquired reflectivity sequence to the unit response of the

Earth medium by a suitable scaling factor.

Since the statistically acquired reflectivity sequence and seismic wavelet appear geophysically reasonable, the blind deconvolution of reflection data is judged as successful. The results we have obtained indicate that we have uncovered the information about the seismic impedances that are coded into the measured seismic traces.

Further work is under way to handle data from other seismic surveys.

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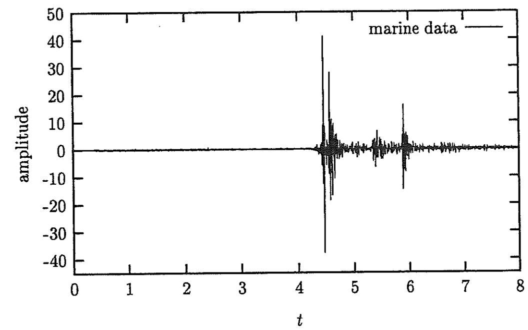

FIG. 1: 90th seismic trace

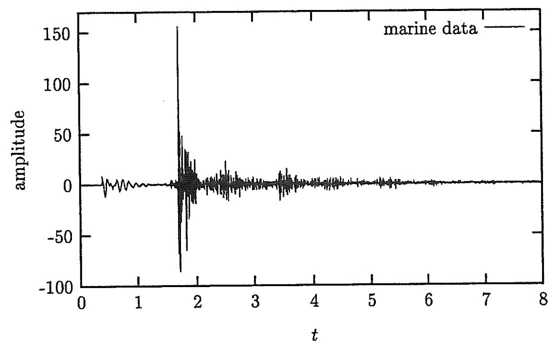

FIG. 2: 240th seismic trace.

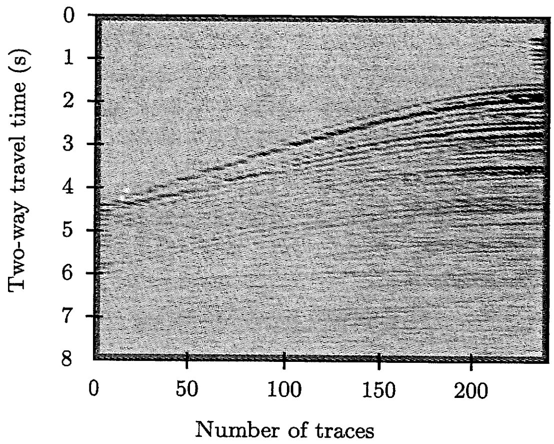

FIG. 3: The seismic data without moveout correction.

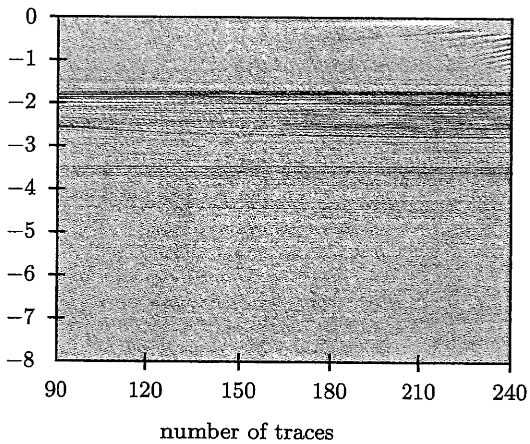

FIG. 4: The seismic data with moveout correction.

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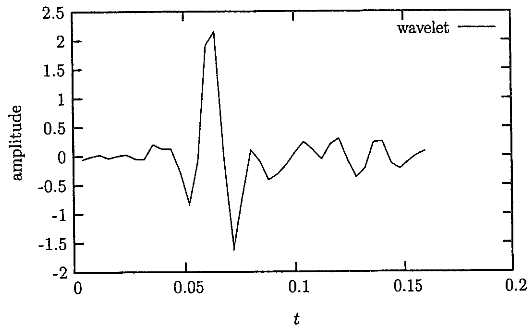

FIG. 5: Seismic wavelet statistically estimated from a dataset in Fig. 4.

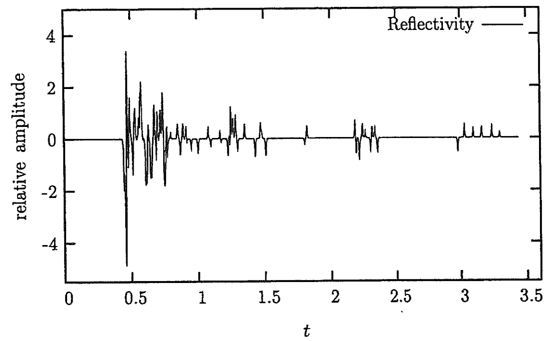

FIG. 6: Statistically estimated reflectivity sequence corresponding to a seismic wavelet in Fig. 5.

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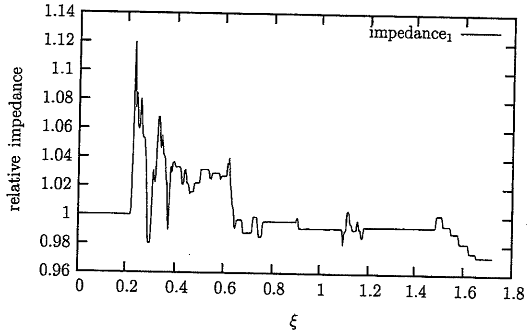

FIG. 7: Estimated seismic impedance corresponding to a statistically estimated reflectivity sequence in Fig. 6.

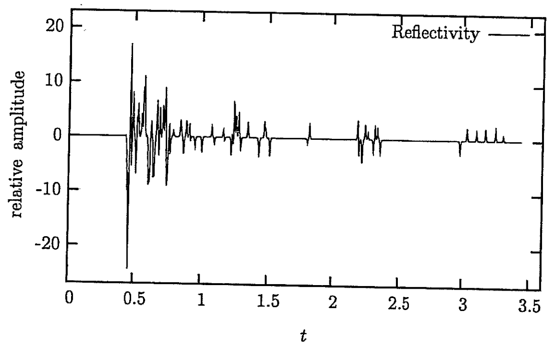

FIG. 8: Scaled reflectivity sequence in Fig. 6.

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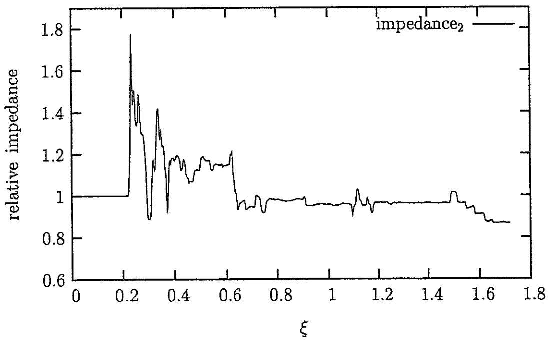

FIG. 9: Estimated seismic impedance corresponding to the scaled reflectivity sequence in Fig. 8.

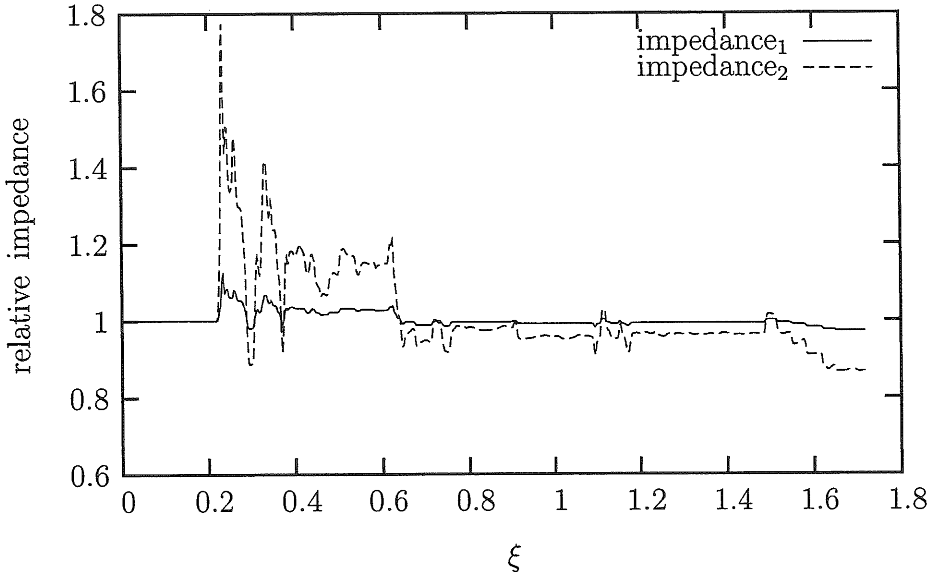

FIG. 10: Comparison between the estimated seismic impedances in Figs. (9) and (7) with and without a scaling factor respectively.