INVERSION OF BOTTOM BACK-SCATTERING MATRIX

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Abstract

The normal mode model of reverberation in shallow-water waveguides has been presented based on Born approximation. The key component of this model is the modal back-scattering matrix. The characteristics of the modal back-scattering caused by (1) the roughness of the bottom interface and (2) the volume inhomogeneities under the interface are discussed. Approaches of inversion of the matrix from reverberation data are proposed. Examples of the inversed result are shown both for numerical simulation and experiment.

1. Introduction

Reverberation process includes sound transmission and scattering. In shallow water the bottom scattering can be considered as the dominant scattering source, especially for a relatively smooth sea surface or for a water sound speed profile with a negative gradient[1]. Therefore, it is important to understand the characteristics and the mechanisms of bottom modal back-scattering in order to establish a predictable model. Bottom scattering, generally, can be attributed to two major mechanisms: (a) roughness of various interfaces; (b) volume inhomogeneities of the medium parameters – density and compressibility. Shallow-water reverberation due to roughness[2] and volume inhomogeneities[3] has been developed separately based on small perturbation theory[4]. Recently, a unified approach to volume and roughness scattering is proposed[5], which provides the possibility of solving the scattering field due to both of the two mechanisms.

The key component for modeling reverberation in shallow-water is the bottom modal back-scattering matrix Θ_{mn}. To extract the Θ_{mn} from the reverberation data has been a challenging topic for a long time. There are some work dealing with the extraction of Θ_{mn} from the reverberation data[6-8]. However, the inversion are based on either empirical law (Lambert' law) or an assumption that Θ_{mn} is separable:

$$\Theta_{mn} = \Theta_m \Theta_n \tag{1}$$

Recently, Shang[9] has proposed a new approach to extract Θ_{mn} from the reverberation data without any a priori assumption on the scattering. With this method, the bottom backscattering matrix Θ_{mn} can be extracted by mode-filtering at the receiving vertical array and changing the point source depth to obtain different incident mode excitation. Some numerical simulations are conducted in [10].

In this paper, the normal mode model of reverberation in shallow-water waveguides has been presented based on Born approximation firstly. Then the characteristics of the modal back-scattering caused by (1) the roughness of the bottom interface and (2) the volume inhomogeneities under the interface are discussed. At last, approaches of inversion of the matrix from reverberation data are proposed. Examples of the inversed result are shown both for numerical simulation and experiment.

2. Normal Mode Model of Reverberation in Shallow-Water Waveguide

The acoustic pressure due to a point source of unit strength at depth z_s in a shallow-water waveguide can be written as

$$p(r,z)=i\pi\sum_{m=1}^{M}\phi_m(z_s)\phi_m(z)H_0^{(1)}(K_m r) \tag{2}$$

where z is the depth coordinate(measured positive downward from the ocean surface), r is the range coordinate, M is the number of trapped modes, ϕ_m are the mode functions, $H_0^{(1)}$ is the Hankel function of zeroth order and first kind, and K_m is the mode wave number including attenuation, $K_m = k_m + i\delta_m$. An implict time dependence exp($-i\omega t$) due to the source at frequency $f = \omega/2\pi$ has been suppressed. By writing the asymptotic form of the Hankel function

$$H_0^{(1)}(K_m r)=\sqrt{2/(\pi k_m r)}\exp\{i(K_m r-\pi/4)\} \tag{3}$$

and using $\exp(i\pi/4)=i^{(1/2)}$, the Eq.(2) can be expressed as

$$p(r,z)=(2\pi i)^{1/2}\sum_{m=1}^{M}\phi_m(z_s)\phi_m(z)\frac{e^{ik_m r}e^{-\delta_m r}}{(k_m r)^{1/2}} \tag{4}$$

In [5], the exact expression of scattering can be presented as multiple scattering series, and the first-order solution is called the single-scattered field or Born approximation. In this paper, the Born approximation of backscattering is considered, and the reverberation field can be represented based on normal mode:

$$p^s(r,z)\approx(\frac{2\pi}{k_0 r})\sum_m\sum_n\phi_m(z_s)T_{mn}\phi_n(z)e^{-(\beta_m+\beta_n)r} \tag{5}$$

where T_{mn} is scattering kernel function, for roughness and medium volume inhomogeneities, it can be expressed as

$$T(k_m,k_n)=\begin{cases} S_{mn}^R\cdot\int_A\gamma(r)e^{i(k_m+k_n)r}dr & (6)\\ S_{mn}^V\cdot\iint_V\varepsilon(r,z)e^{i(k_m+k_n)r-(\xi_m+\xi_n)z}drdz & (7)\end{cases}$$

where γ describe the interface roughness, and ε describe the volume inhomogeneities. The expression of T_{mn} for roughness[2] is

$$T_{mn}^2=\Theta_{mn}=(\pi/4)\sigma^2\tau_0^{-2}\sin^2\theta_m\sin^2\theta_n G_{mn} \tag{8}$$

where σ is mean square root of roughness, τ_0 is correlation length of the roughness, θ is grazing angle of the normal mode. G_{mn} is proportional to the two-dimensional space spectrum of γ.

And the expression of T_{mn} for volume inhomogeneities[3] is

$$T_{mn}^2=\Theta_{mn}=c_{mn}\cdot L_{mn}\cdot J_{mn} \tag{9}$$

Where

$$c_{mn}=(\frac{\sigma_c}{4\pi\gamma})^2T_m^2T_n^2\cdot[2k_1^2+\eta(k_1^2+\xi_m\xi_n-\beta_m\beta_n)]^2 \tag{10}$$

$$L_{mn}\approx(\pi\ell_h^2)e^{-(\frac{\ell_h}{2})^2(\xi_m+\xi_n)^2} \tag{11}$$

$$J_{mn} = \frac{\beta_m + \beta_n + \ell_v^{-1}}{(\beta_m + \beta_n + \ell_v^{-1})^2 (\beta_m + \beta_n)} \tag{12}$$

where σ_c is the standard deviation of sound speed fluctuation, γ is the density ratio between the bottom and the water column at the source depth, η is a constant, ξ is the eigenvalue of the normal mode, β is the transmission coefficient of a plane wave, L_{mn} is spectrum of the horizontal correlation function of inhomogeneity, J_{mn} is spectrum of the vertical correlation function of inhomogeneity

3. Characteristics of Modal Back-Scattering Matrix

Modal back-scattering matrix is key component of normal reverberation model, and it describes the coupling relationship between incident normal mode and scattering normal mode in the shallow water. In this section, the theory and numerical analysis of back-scattering matrix was given in three aspects: 1) Separable approximation, 2) Sub-Matrix approximation, and 3) Mode-space.

3.1. *Separability of back-scattering matrix*

The modal back-scattering matrix in the shallow water can be expressed as:

$$\Theta^1 = \begin{bmatrix} \Theta_{11} & \Theta_{12} & \cdots & \Theta_{1M} \\ \Theta_{21} & \Theta_{22} & \cdots & \Theta_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \Theta_{M1} & \Theta_{M2} & \cdots & \Theta_{MM} \end{bmatrix} \tag{13}$$

If the matrix is separable, just like Eq.(1), Eq.(8) can become separable matrix

$$\Theta^2 = \begin{bmatrix} \Theta_{11} & \sqrt{\Theta_{11}\Theta_{22}} & \cdots & \sqrt{\Theta_{11}\Theta_{MM}} \\ \sqrt{\Theta_{22}\Theta_{11}} & \Theta_{22} & \cdots & \sqrt{\Theta_{22}\Theta_{MM}} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{\Theta_{MM}\Theta_{11}} & \sqrt{\Theta_{MM}\Theta_{22}} & \cdots & \Theta_{MM} \end{bmatrix} \tag{14}$$

The difference between the full back-scattering matrix(Θ^1) and the separable matrix(Θ^2) is the separability error. Whether the matrix is separable can be decided by quantity of separability error.

As a numerical simulation example, we consider a Pekeris waveguide with water depth $H = 50$m, $c_0 = 1500$m/s, $c_b = 1600$m/s, $\rho_b = 1.77$, bottom attenuation $\alpha = 0.23\lambda$ /dB.

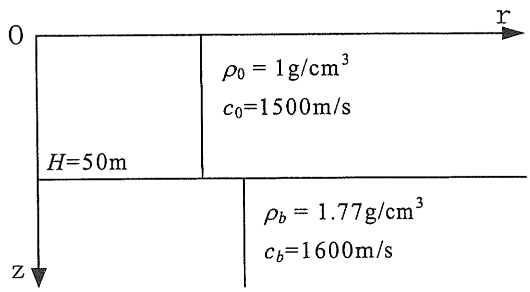

Fig.1. Pekeris waveguide.

Firstly, considering roughness back-scattering. Taking the roughness parameters as: $\sigma = 0.1\text{m}$, $\tau_0 = 6\text{m}$

Then full back-scattering matrix is (dB)

$$\Theta_R^1 = \begin{bmatrix} -52.9126 & -48.1497 & -46.2195 & -45.3982 \\ -48.1497 & -43.3745 & -41.4262 & -40.5837 \\ -46.2195 & -41.4262 & -39.4511 & -38.5774 \\ -45.3982 & -40.5837 & -38.5774 & -37.6677 \end{bmatrix} \tag{15}$$

The separable matrix is (dB)

$$\Theta_R^2 = \begin{bmatrix} -52.9126 & -48.1435 & -46.1818 & -45.2901 \\ -48.1435 & -43.3745 & -41.4128 & -40.5211 \\ -46.1818 & -41.4128 & -39.4511 & -38.5594 \\ -45.2901 & -40.5211 & -38.5594 & -37.6677 \end{bmatrix} \tag{16}$$

Separability error matrix is (dB)

$$\Theta_R^3 = \begin{bmatrix} 0 & 0.0061 & 0.0376 & 0.1081 \\ 0.0061 & 0 & 0.0134 & 0.0626 \\ 0.0376 & 0.0134 & 0 & 0.0180 \\ 0.1081 & 0.0626 & 0.0180 & 0 \end{bmatrix} \tag{17}$$

Then, considering volume inhomogeneity back-scattering. Taking the volume inhomogeneity parameters as: $\sigma_c = 0.025$, $\eta = 5.2$, $l_h = 2\text{m}$, $l_v = 0.5\text{m}$

Then full back-scattering matrix is (dB)

$$\Theta_V^1 = \begin{bmatrix} -86.1092 & -80.9139 & -78.2050 & -78.2752 \\ -80.9139 & -75.7099 & -72.9776 & -72.8896 \\ -78.2050 & -72.9776 & -70.1820 & -69.6368 \\ -78.2752 & -72.8896 & -69.6368 & -61.4255 \end{bmatrix} \tag{18}$$

The separable matrix is (dB)

$$\Theta_V^2 = \begin{bmatrix} -86.1092 & -80.9095 & -78.1456 & -73.7674 \\ -80.9095 & -75.7099 & -72.9459 & -68.5677 \\ -78.1456 & -72.9459 & -70.1820 & -65.8037 \\ -73.7674 & -68.5677 & -65.8037 & -61.4255 \end{bmatrix} \tag{19}$$

Separability error matrix is (dB)

$$\Theta_V^3 = \begin{bmatrix} 0 & 0.0043 & 0.0594 & 4.5079 \\ 0.0043 & 0 & 0.0317 & 4.3220 \\ 0.0594 & 0.0317 & 0 & 3.8331 \\ 4.5079 & 4.3220 & 3.8331 & 0 \end{bmatrix} \tag{20}$$

From Eq.(17) and Eq.(20), we can decided that the back-scattering matrix due to roughness is quasi-separable; and the back-scattering matrix due to inhomogeneities is unseparable.

3.2. *Sub-matrix of back-scattering matrix*

In Eq.(15) and Eq.(18), the elements of right-down is bigger than the elements of left-up of the matrices. In this section, the right-down submatrix is used to synthesize reverberation data in stead of full back-scattering matrix.

As a numerical simulation example, we consider the same Pekeris waveguide as we used in section 3.1. Considering center frequency is f = 450Hz, so there are 11 trapped normal modes in the waveguide.

Taken the order of submatrix as 11, 10, 9 and 8 respectively, The synthesized reverberation data used four submatrices are in the Fig.2

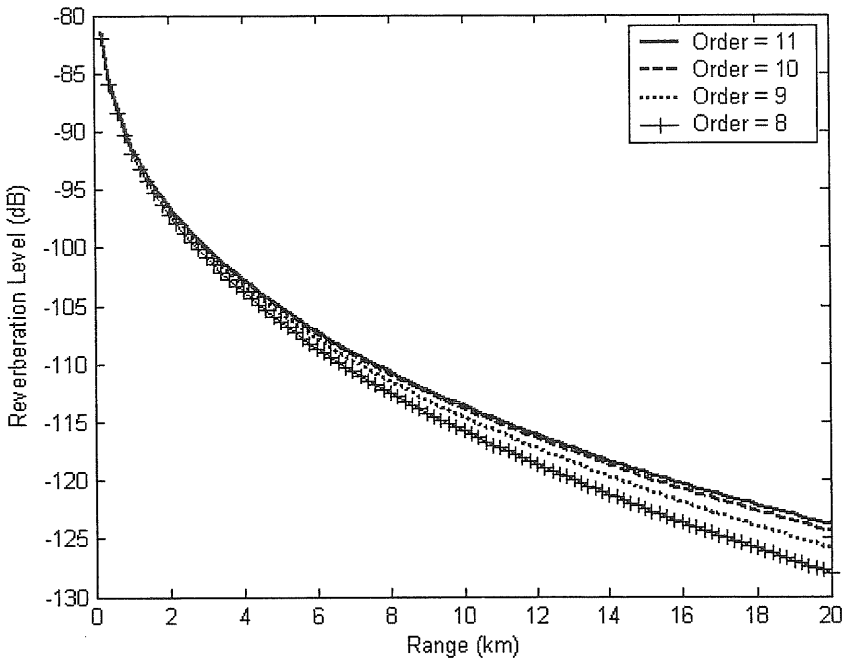

Fig.2. Reverberation data using submatrices.

The full modal back-scattering matrix can be replaced by submatrix sometimes judged from the Fig.2.

3.3. *Characteristics of back-scattering matrix in mode space*

For convenience, we define the "effective matrix" $\Theta_{mn}{}^{eff}$ as follows

$$\Theta_{mn}^{eff} = \Theta_{mn} \exp\{-2(\delta_m + \delta_n) r_c\} \tag{21}$$

In the numerical simulation, we consider the same Pekeris waveguide as section 3.1. Considering centre frequency f = 150Hz, so there are 4 trapped normal modes in the waveguide. We only consider the diagonal elements of the effective matrix because it is quasi-separable for roughness back-scattering. The ranges are 500m, 5km, 10km and 20km respectively.

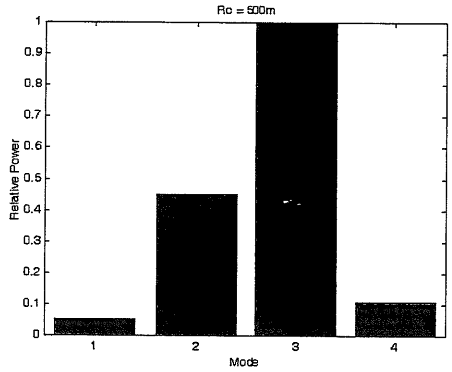

Fig.3. $r_c = 500$ m.

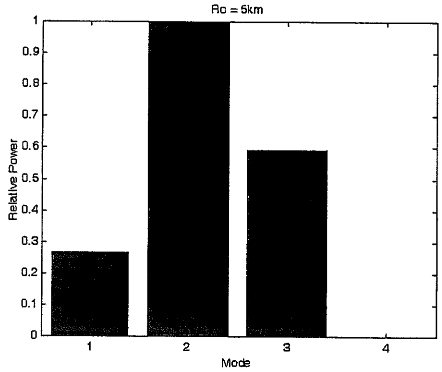

Fig.4. $r_c = 5$ km.

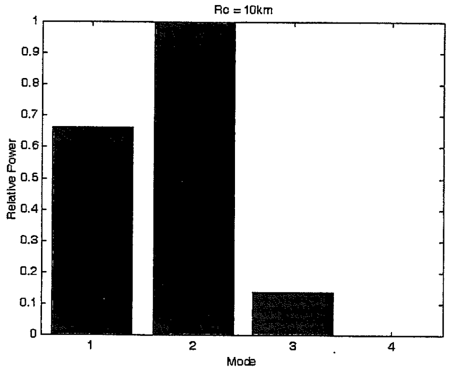

Fig.5. $r_c = 10$ km.

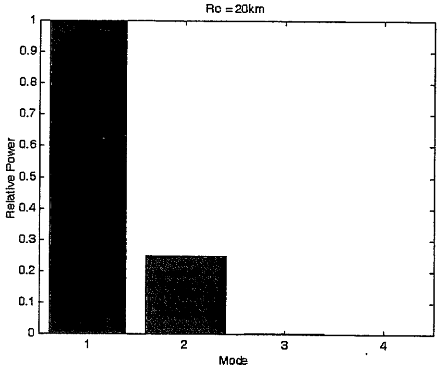

Fig.6. r_c =20 km.

The results(Fig.3-Fig.6) show that: 1) in near distance, the value of back- scattering matrix elements inclines towards high modes. The matrix is similar to highpass filter; 2) in middle distance, the value of back-scattering matrix elements inclines towards middle modes. The matrix is similar to bandpass filter; 3) in far distance, the value of back-scattering matrix elements inclines towards low modes. The matrix is similar to lowpass filter. This phenomenon is caused by mode attenuation in shallow water waveguide.

4. Inversion of Modal Back-Scattering Matrix

Extracting the bottom back-scattering information from reverberation data in shallow-water waveguide is an attractive but difficult issue. In this section, two kinds of methods were proposed. One has a priori assumption; the other has no a priori assumption. They were called separable inversion and unseparable inversion of modal back-scattering matrix[11].

4.1 Numerical simulation of inversion of modal back-scattering matrix

First, make an assumption that the modal back-scattering matrix Θ_{mn} is separable (Eq.1)

Then, make mode-filtering of the reverberation field. When the receiving vertical array is weighted by the j-th mode function, we have

$$p_j^s(z_0,r_c) = \int p^s(z,z_0,r_c)\phi_j(z)dz_r$$

$$= \frac{p_0}{k_0 r}\sum_n^M \phi_n(z_0)\phi_n(z_H)\cdot S_{nj}\cdot\phi_j(z_H)\int_A \eta(x)e^{i(\xi_n+\xi_j)x}dx \qquad (22)$$

Taking only the non-interference term of the reverberation intensity as the averaged intersity

$$I_j(z_0,r_c) = \frac{I_0}{k_0^2 r_c}\sum_n^M \phi_n^2(z_0)\Theta_{nj}\exp\{-2(\delta_n+\delta_j)r_c\} \qquad (23)$$

Let $j = 1, 2$

$$\frac{I_1}{I_2} = \frac{\exp(-2\delta_1 r_c)\Theta_1}{\exp(-2\delta_2 r_c)\Theta_2} \qquad (24)$$

Eq.(24) can be transformed to the following equation

$$\Theta_2 = (\frac{I_2}{I_1})(\frac{\exp(-2\delta_1 r_c)}{\exp(-2\delta_2 r_c)})\Theta_1 \qquad (25)$$

And

$$\Theta_3 = (\frac{I_3}{I_1})(\frac{\exp(-2\delta_1 r_c)}{\exp(-2\delta_3 r_c)})\Theta_1 \qquad (26)$$

......

$$\Theta_M = (\frac{I_M}{I_1})(\frac{\exp(-2\delta_1 r_c)}{\exp(-2\delta_M r_c)})\Theta_1 \qquad (27)$$

Integrated (25),(26),(27) and (23), we can get Θ_1、Θ_2、Θ_3、$\cdots$ and Θ_M. At last, get the modal back-scattering matrix Θ_{mn} using the separability(Eq.1).

Neglecting the dispersion effect, we have

$$I_j(z_0,r_c) = \frac{I_0}{k_0^2 r_c}\sum_m^M \phi_m^2(z_0)\Theta_{mj}\exp\{-2(\delta_m+\delta_j)r_c\} \qquad (28)$$

In Eq.(28), we have assumed that the unperturbed stratified waveguide is known, which means that Θ_{mj} is the only unknown.

If, for each filtered modal reverberation intensity I_j, we change the source depth z_s M times: $z_{s1}, z_{s2}, \cdots, z_{sM}$, then we will have $M\times M$ equations for solving the $M\times M$ unknown Θ_{mj}

In the numerical simulation, we consider the same Pekeris waveguide as section 3.1. And take the roughness back-scattering as an example, its roughness parameters is: σ = 0.1m, τ_0 = 6m

The back-scattering matrix used to synthesize reverberation data is Eq.(15). And the inversed back-scattering matrix using separable method is

$$\Theta_R^{sep} = \begin{bmatrix} -52.9640 & -48.1760 & -46.2087 & -45.3440 \\ -48.1760 & -43.3880 & -41.4207 & -40.5560 \\ -46.2087 & -41.4207 & -39.4533 & -38.5886 \\ -45.3440 & -40.5560 & -38.5886 & -37.7239 \end{bmatrix} \qquad (29)$$

The inversed back-scattering matrix using unseparable method is

$$\Theta_R^{unsep} = \begin{bmatrix} -52.9126 & -48.1497 & -46.2195 & -45.3982 \\ -48.1497 & -43.3745 & -41.4262 & -40.5837 \\ -46.2195 & -41.4262 & -39.4511 & -38.5774 \\ -45.3982 & -40.5837 & -38.5774 & -37.6677 \end{bmatrix} \tag{30}$$

At last, we get the separable inversion error and unseparable inversion error. The separable inversion error is (dB)

$$|\Theta_R^{sep} - \Theta_R^1| = \begin{bmatrix} 0.0515 & 0.0264 & 0.0108 & 0.0542 \\ 0.0264 & 0.0135 & 0.0055 & 0.0277 \\ 0.0108 & 0.0055 & 0.0022 & 0.0112 \\ 0.0542 & 0.0277 & 0.0112 & 0.0562 \end{bmatrix} \tag{31}$$

The unseparable inversion error is (dB)

$$|\Theta_R^{unsep} - \Theta_R^1| = \begin{bmatrix} 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 \end{bmatrix} \tag{32}$$

4.2. *Inversion of modal back-scattering matrix from experiment data*

Reverberation data were collected in a reasonably flat shallow-water area in South China Sea. A vertical line array was deployed to record monostatic reverberation from explosive charges. The explosive charges denoted at a depth of 7m. The vertical array contained 32 hydrophones, which were spaced from 7m to 69m. The sound speed profile measured during experiment is shown in Fig.7. The depth of the experiment sea area is 88.84m.

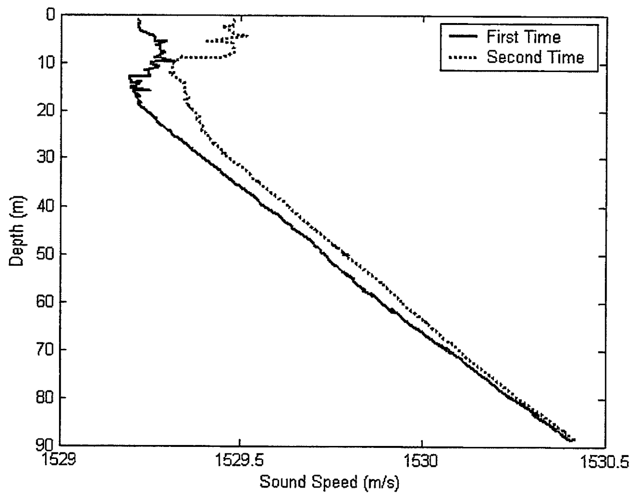

Fig.7. Sound speed profile.

Firstly, the separable inversion of the back-scattering matrix was presented. The separable inversion include following six steps:

The first step: make frequency filtering of the reverberation data, the center frequency is $f = 200$Hz

The second step: make mode-filtering of the reverberation data

The third step: get the $I_j(j=1,2,\cdots,M)$

The fourth step: get the ratio of $\Theta_m(m=1,2,\cdots,M)$

The fifth step: calculate the Θ_1

The sixth step: get the back-scattering matrix

$$\Theta^{sep} = \begin{bmatrix} -82.9903 & -82.6730 & -82.3185 & -76.6115 & -75.0499 & -65.9585 \\ -82.6730 & -82.3557 & -82.0012 & -76.2942 & -74.7326 & -65.6412 \\ -82.3185 & -82.0012 & -81.6466 & -75.9397 & -74.3781 & -65.2867 \\ -76.6115 & -76.2942 & -75.9397 & -70.2327 & -68.6712 & -59.5797 \\ -75.0499 & -74.7326 & -74.3781 & -68.6712 & -67.1096 & -58.0182 \\ -65.9585 & -65.6412 & -65.2867 & -59.5797 & -58.0182 & -48.9268 \end{bmatrix} \tag{33}$$

Then, the unseparable inversion of the back-scattering matrix was discussed. The reverberation intensity can expressed as:

$$I(z_0, z, r_c) = \frac{I_0}{k_0^2 r_c} \sum_m^M \sum_n^M \phi_m^2(z_0)\Theta_{mn}\phi_n^2(z)\exp\{-2(\delta_m + \delta_n)r_c\} \tag{34}$$

The number of trapped mode in the waveguide is M=6, the unkown parameters the back-matrix is M^2-(1/2)(M^2-M)=21

$$\Theta = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \\ x_2 & x_7 & x_8 & x_9 & x_{10} & x_{11} \\ x_3 & x_8 & x_{12} & x_{13} & x_{14} & x_{15} \\ x_4 & x_9 & x_{13} & x_{16} & x_{17} & x_{18} \\ x_5 & x_{10} & x_{14} & x_{17} & x_{19} & x_{20} \\ x_6 & x_{11} & x_{15} & x_{18} & x_{20} & x_{21} \end{bmatrix} \tag{35}$$

Using 21 channels of receiver array, we can get the unseparable back-scattering matrix

$$\Theta^{unsep} = \begin{bmatrix} -84.9205 & -78.9090 & -81.4483 & -80.5997 & -77.7279 & -67.0755 \\ -78.9090 & -76.6619 & -77.6056 & -73.0682 & -67.7421 & -56.3842 \\ -81.4483 & -77.6056 & -74.9338 & -73.0582 & -80.4350 & -60.9133 \\ -80.5997 & -73.0682 & -73.0582 & -72.9733 & -71.1268 & -53.2428 \\ -77.7279 & -67.7421 & -80.4350 & -71.1268 & -71.2151 & -51.7836 \\ -67.0755 & -56.3842 & -60.9133 & -53.2428 & -51.7836 & -46.2848 \end{bmatrix} \tag{36}$$

5. Conclusion

In this paper, the normal mode model of reverberation in shallow-water waveguides has been presented based on Born approximation. Then characteristics of the modal back-scattering caused by (1) the roughness of the bottom interface and (2) the volume inhomogeneities under the interface are discussed. The back-scattering matrix due to roughness is quasi-separable; and the back-scattering matrix due to inhomogeneities is unseparable. In the certain condition, the full back-scattering matrix can be replaced by its submatrix. In mode-space, 1) in near distance, the value of back- scattering matrix elements inclines towards high modes. The matrix is similar to highpass filter; 2) in middle distance, the value of back-scattering matrix elements inclines towards middle modes. The matrix is similar to bandpass filter; 3) in far distance, the value of back-scattering matrix elements inclines towards low modes. The matrix is similar to lowpass filter. This phenomenon is caused by mode attenuation in shallow water waveguide.

Two approaches of inversion of the matrix from reverberation data are proposed. One is separable inversion method, the other is unseparable method. Examples of the inversed result of the two inversion methods are shown both for numerical simulation and experiment.

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