ESTIMATION OF SHEAR WAVE VELOCITY IN SEAFLOOR SEDIMENT BY SEISMO-ACOUSTIC INTERFACE WAVES: A CASE STUDY FOR GEOTECHNICAL APPLICATION

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Estimates of shear wave velocity profiles in seafloor sediments can be obtained from inversion of measured dispersion relations of seismo-acoustic interface waves propagating along the seabed. The interface wave velocity is directly related to shear wave velocity with value of between 87-96% of the shear wave velocity, dependent on the Poission ratio of the sediments. In this paper we present two different techniques to determine the dispersion relation: a single-sensor method used to determine group velocity and a multi-sensor method used to determine the phase velocity of the interface wave. An inversion technique is used to determine shear wave velocity versus depth and it is based on singular value decomposition and regularization theory. The technique is applied to data acquired at Steinbåen outside Horten in the Oslofjorden (Norway) and compared with the result from independent core measurements taken at the same location. The results show good agreement between the two ways of determining shear wave velocity.

1 Introduction

The structure and composition of the seabed's structure are very important for many applications. For evaluation of long range sonar performance it is necessary to have precise information about the layered structure of the seabed with the densities, sound speeds and attenuations.

Quantitative characterization of the upper part of the seabed is also of major importance in both for the geotechnical and offshore industry. To reduce the risk and cost associated to sea-bottom installations such as communication cables, gas/oil cables and underwater constructions, precise and reliable information about the seafloor is needed. For this purpose it is important to know the seismo-acoustic parameters such as compressional P-wave and shear S-wave velocity, density and attenuation as function of depth. Shear wave velocity is in this context unique since it is related to shear strength of the sediments and hence used to evaluate how much load the seabed can support.

In some cases the geoacoustic properties can be acquired by *in-situ* measurement, or by taking samples of the bottom material with subsequent measurement in laboratories. In practice this direct approach is often not sufficient and will have to be supplemented by information acquired by remote measurement techniques in order to have the area coverage and the depth resolution required.

A possible approach for measuring the seabed's shear wave structure is to use seismo-acoustic interface waves, also known as Rayleigh, Stoneley, or Scholte waves, that may exist at an interface between two media, at least one of which must be a solid.

References to this technique are the papers by Caiti, Stoll and Akal [2], Jensen and Schmidt [7] and Rauch [11].

A general property of interface waves is that they propagate along the interface with a velocity that is closely related to the shear wave velocity. The phase velocity of an interface wave in a homogeneous solid half-space is dominated by the shear wave velocity v_s. and varies from 0.87 v_s to 0.96 v_s, depending on the Poission ratio of the medium. The amplitude decays exponentially with distance away from the interface; the penetration is typically at the order of one wavelength [12]. If the shear wave velocity varies with depth in the bottom, which is normally this case, the interface wave velocity becomes dependent on the frequency, i.e. the interface waves are in general dispersive and the dispersion is given by the shear velocity profile of the bottom.

The objective of the study reported in this paper is to compare the shear wave velocity values obtained by analyzing the dispersion characteristics of recorded interface waves, and to compare these results with the results using common geotechnical techniques. Therefore we conducted, in 1998, a joint seismo-acoustic and geotechnical experiment at a location called Steinbåen outside Horten in the Oslofjorden (Norway). The seismo-acoustic data was acquired by the company Geomap AS and the geotechnical investigations were done by FUGRO LTD [5, 6], both companies working under a contract with Norwegian University of Science and Technology as part of the European MAST III project *ISACS*.

2 Data collection

In 1998 a modified refraction seismic survey was conducted by Geomap AS at Steinbåen outside Horten in Norway. The water depth at the location is 18 m. A 34.5 meter long linear hydrophone array with 24 hydrophones with spacing of 1.5 meter was used for the recording, and small dynamite charges were used as source, see figure 1. The difference between this setup and normal refraction seismic survey was that the recording length in time was increased from 1 second to 8 seconds. This was done to ensure that the seismo-acoustic interface waves were captured, due to their slow propagation speed compared to the speed of the compressional waves. The sources were set to explode approximately of 77 meter in front of receiver no 1 in the hydrophone array.

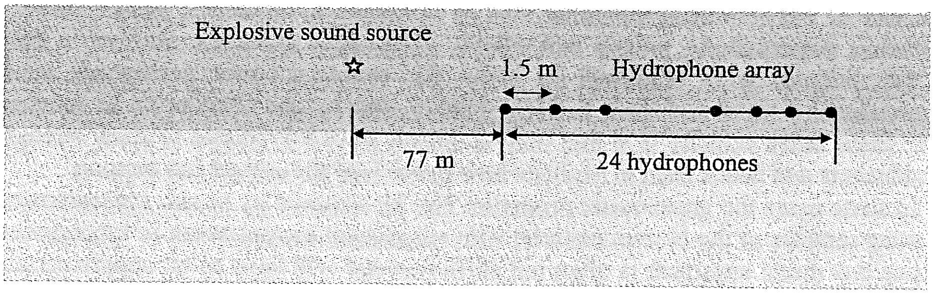

Figure 1. Experimental setup for reception of interface waves by a 24-hydrophone array with spacing 1.5 m situated on the seabed. The distance between source and receiver no 1 is 77 m.

The recorded acoustic/seismic data were analyzed with the aim of determining both the compressional (P) and shear (S) wave velocity. The refracted compressional wave velocity was determined to be 1515 m/s in the upper part of the sediment. From data we have indication of a deeper and harder layer with compressional wave velocity of about 2500 m/s, but it was not possible to determine the exact depth of this layer.

A short time before this survey FUGRO LTD had done a complete standard geotechnical investigation and gathered information from box cores, gravity cores and Piezocone penetration (PCZT) for the purpose to characterize and classify the sediments of the upper seabed at the site. The depth of the gravity cores was about 3.5 m in the seabed, but due to the soft seabed it was difficult to get good cores for later laboratory testing. Therefore only two core samples were recovered taken to a laboratory where the shear strength and the shear wave velocity were measured using the resonance column method. The result from the core analysis showed that the shear wave velocity was 95 m/s in the interval from 2.4 m to 2.6 m and 94.7 m/s at 2.6m to 2.8 m. The density of the sediment was about 1780 kg/m3. The geotechnical description of the sediment is "very soft olive grey clay with occasional fine gravel from 0-0.1m, slightly sandy down to 0.5m and occasionally shell fragments below 0.5m. The water content is about 40 %".

3 Estimation of interface wave dispersion

In this paper two different methods are used to extract the dispersion characteristics of the interface waves.

The multi-sensor method uses all the 24 traces simultaneously [4, 9] and the method of Principal Components Decomposition for the determination of the locations of the spectral lines in the wave number spectra [1]. These values are then transformed to estimates of the phase velocity by using the known element spacing. The method assumes that seabed parameters are locally range-independent in the range interval covered by the hydrophone array. The method does not need to know the distance between source and receiver, only the sensor spacing need to be known.

The single-sensor method estimates the group velocity using the Wavelet Transform (WT) [10] applied to one single trace (hydrophone) at the time [13]. The method requires that the distance between source and the receiver is known. The main advantage of the single-sensor method is that it can be used to study velocity variations with range [8]. The Wavelet analysis can be viewed as a multiple filter technique, and has much in common with the Gabor matrix method [3] used for instance by Caiti et al. [2]. However, there are important differences. While the Gabor matrix has fixed bandwidth filters, the Wavelet Transform has a continuously varying filter bandwidth. This is an advantage that improves the time-frequency resolution of the processed data, which gain improves the discrimination of the different modes of the dispersion curve. According to the uncertainty principle, the spectral components of a signal cannot be located exactly in both time and frequency. With WT method, narrow band pass filters are used to detect the low frequencies components and wide band pass filters to detect the high frequencies. In this way, the WT method gives good frequency resolution and poor time resolution at low frequencies, and good time resolution and poor frequency resolution at high frequencies.

4 Inversion algorithm

The inversion algorithm used in this study is a modified version of the inversion algorithm reported by Caiti et al. [2]. The main part in the inversion algorithm is the singular value decomposition (SVD) of a linear system and a forward model to determine dispersion curves from the "earth model". Assuming that the thickness and compressional wave velocity for each layer are known and with fixed parameters, the model is able to generate a group/phase velocity vector $\mathbf{v}_c \in R^n$ as a function of the shear velocity vector $\mathbf{v}_s \in R^m$:

$$T\mathbf{v}_s = \mathbf{v}_c , \tag{1}$$

where Jacobian $T \in R^n \times R^m$. Here, m is the index over layers and n is the index over frequencies of the dispersion curve, and we will consider the most common case where $m \leq n$; that is, we have more data than estimated parameters. The solution, which is expressed as

$$\mathbf{v}_s = (T^T T)^{-1} T^T \mathbf{v}_c \tag{2}$$

minimizes $||T\mathbf{v}_s - \mathbf{v}_c||^2$ *in the least square sense* and here $||\cdot||$ is the 2 norm of a vector. By using the singular value decomposition (SVD) to the rectangular matrix T the solution can be expressed as

$$\mathbf{v}_s = W\Omega^{-1} U^T \mathbf{v}_c , \tag{3}$$

$$\mathbf{v}_s = \sum_{i=1}^{m} \frac{(\mathbf{u}_i^T \mathbf{v}_c)}{\omega_i} \mathbf{w}_i = \sum_{i=1}^{m} \frac{\alpha_i}{\omega_i} \mathbf{w}_i . \tag{4}$$

In equations (2) and (3) $T^T = W[\Omega \ \mathrm{O}]U^T$, U and W are unitary orthogonal matrices with dimension ($n \times n$) and ($m \times m$) respectively and Ω is a square diagonal matrix of dimension m, with diagonal entries ω_i, called singular values of T with $\omega_1 > \omega_2 > \ldots > \omega_m$; O is a zero matrix with dimension ($m \times (n-m)$); $\mathbf{u}_i$ is the ith column of U and $\mathbf{w}_j$ the jth column of W. There is an effect of ill-conditioning in the numerical solution of this inverse problem and regularization theory is used to reduce the effect with the modified normal equation

$$\mathbf{v}_s^* = (T^T T + \gamma H^T H)^{-1} T^T \mathbf{v}_c . \tag{5}$$

where H, a square matrix with dimension ($m \times m$), is a general operator embedding the *a priori* constraints imposed on the solution, and the regularization parameter $\gamma > 0$. The regularized solution is given by

$$\mathbf{v}_s^* = T^\dagger \mathbf{v}_c \tag{6}$$

with

$$T^\dagger = W(\Omega + \Omega^{-1}\gamma (HW)^T (HW))^{-1} U^T \tag{7}$$

In this practical case, we have used $H = I$, unit matrix, and hence $T^\dagger = W(\Omega + \gamma\Omega^{-1})^{-1} U^T$.

Figure 2 shows a flowchart of the inversion algorithm. The input data consist of the measured dispersion curves from the previous processing described above, and the structure of an earth model. This earth model has a given number of sediment layers; the parameters are the compressional and shear wave velocities and the densities. In the current application the layer thickness, here is 2 meters, the compressional velocities and the densities are fixed; only the shear wave velocities of the layers are assumed unknown at the initialization. The parameters of the earth model are inputted to a forward acoustic model (based on the scheme by Takeuchi H. et al [12]) which calculates a synthetic dispersion curve. The measured and synthetic dispersion curves are compared by an objective function, $\phi(v_s) = ||Tv_s - v_c||^2$ and the objective function is minimized by varying the free earth model parameters, in this case the shear wave velocities of the layers. This iteration is continued until an acceptable fit between the measured and the synthetic dispersion curve is obtained. The results are estimates of the shear wave velocities of the sediment layers together with an uncertainty analysis.

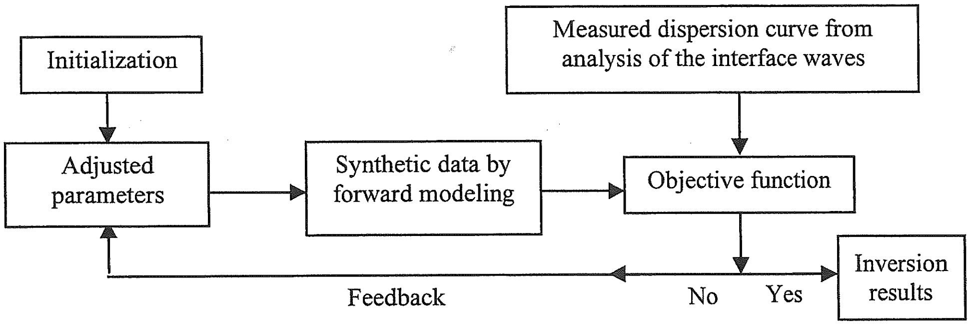

Figure 2. Flowchart of the inversion algorithm with five parts: initial environmental parameters' setting, acoustic forward modeling, comparison of the synthetic data computed by forward acoustic model and observed data, here the dispersion curve of the group/phase velocity extracted from the interface waves, by an objective function, minimizing the objective function by varying the environmental parameters and uncertainty analysis for inversion results, here shear wave velocity.

5 Data analysis

Figure 3 shows in the left panel the time signal traces recorded on the 24 hydrophone array from one particular shot P1-86 at the Steinbåen site. The left panel shows the raw data with the full frequency bandwidth. The middle panel shows the high pass filtered and zoomed version of the same traces, the high frequency filtering emphasizes the refracted arrivals and we can observe an refracted arrival which determines the compressional wave velocity of the upper sediment layer to approximately 1515 m/s. In the right panel the raw data have been low pass filtered, which brings out the interface waves. The amplitudes of the interface waves are weak compared to the water borne modes (compare left and right panels in Fig 3), but one can clearly observe that the velocity of the interface waves is in the range of 40 to 100 m/s

The low pass filtered versions of each of the recorded traces are used in the subsequent dispersion analysis using the time-frequency analysis of the Wavelet Transform (WT), and time is converted to velocity since the distance between source and the receiver is known. An example of the analysis of trace no 10 is shown in figure 4 as group velocity as function of frequency. Both panels show the same results but presented in two different ways. The left panel shows a colormap and in the right panel the data are plotted in contour plot. From the latter representation it is easy to detect the maximum values along the each contour as indicated by the stars in the left panel of figure 4. The detected maximum values are the samples of the dispersion curve that will be used as the measured data in the inversion algorithm of figure 2.

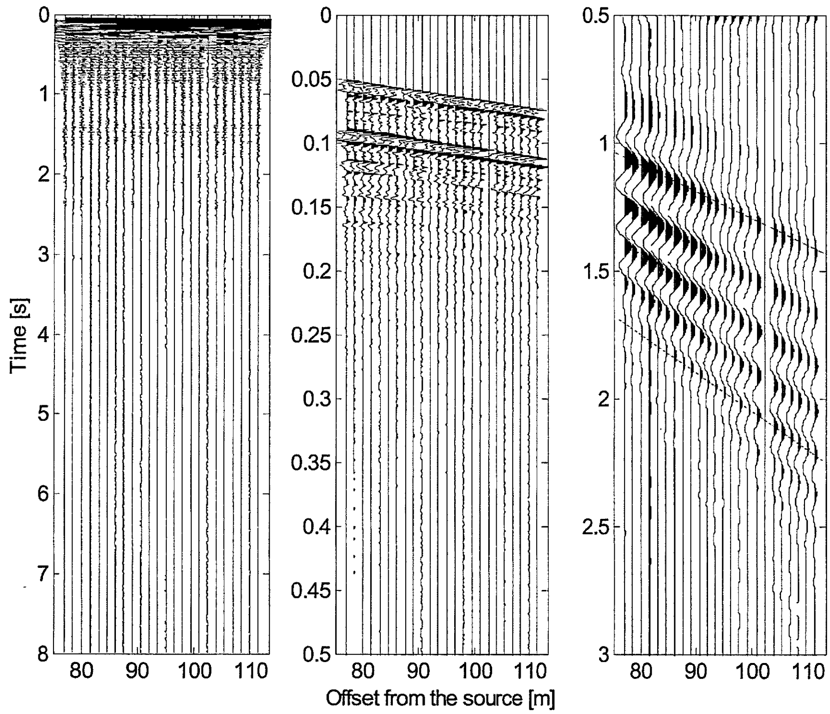

Figure 3. Time signal traces recorded on the 24 hydrophone array from shot P1-86 in Steinbåen site. Left panel: The raw data with full bandwidth; Middle panel: High pass filtered traces emphasizing the refracted arrivals and Right panel: Low pass filtered traces brings out the interface wave. A simple interpretation of the middle panel indicates that the P-wave velocity of the upper part of the seabed is about 1510 m/s. The dispersion of the interface wave in the right panel indicates that the velocity of the interface waves is in the range of 40 to 100 m/s.

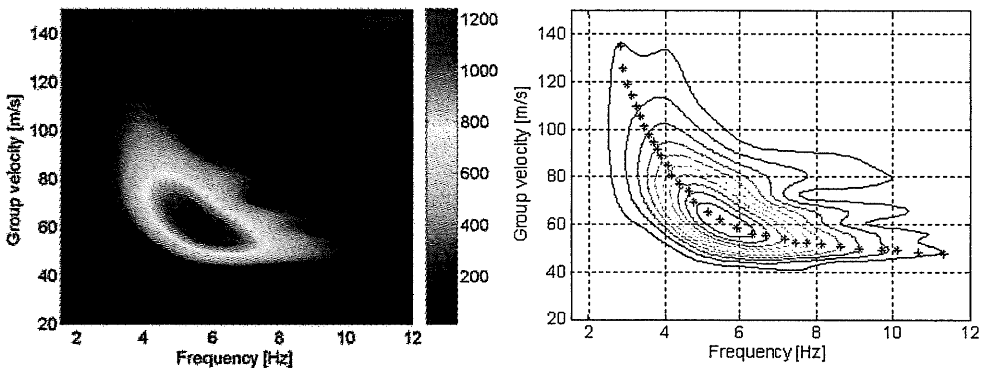

Figure 4. Dispersion analysis of trace-10 from shot P1-86 in Steinbåen site. Left panel: Dispersion color map of group velocity; Right panel: Same dispersion map but in contour plot and the dispersion curve (stars) is identified by picking the maximum value along the each contour.

Figure 5 shows the results of the inversion. The results are presented in four panels. The top left panel shows the group velocity data extracted from the dispersion contour map of the right panel in figure 4 (stars) and the model fit as a continuous line. The bottom left panel shows the resulting shear wave velocity versus depth (blue thick line) with error estimates (red thin line). The error estimate was generated assuming an uncertainty of 15m/s in the group velocity pick. The upper right panel shows the singular values of the SVD of the Jacobian matrix *T*. The singular values to the left of the blue thick line are larger than the regularization parameter γ. The corresponding singular vectors that constitute the resulting shear wave velocity profile are marked with blue color in the bottom right panel.

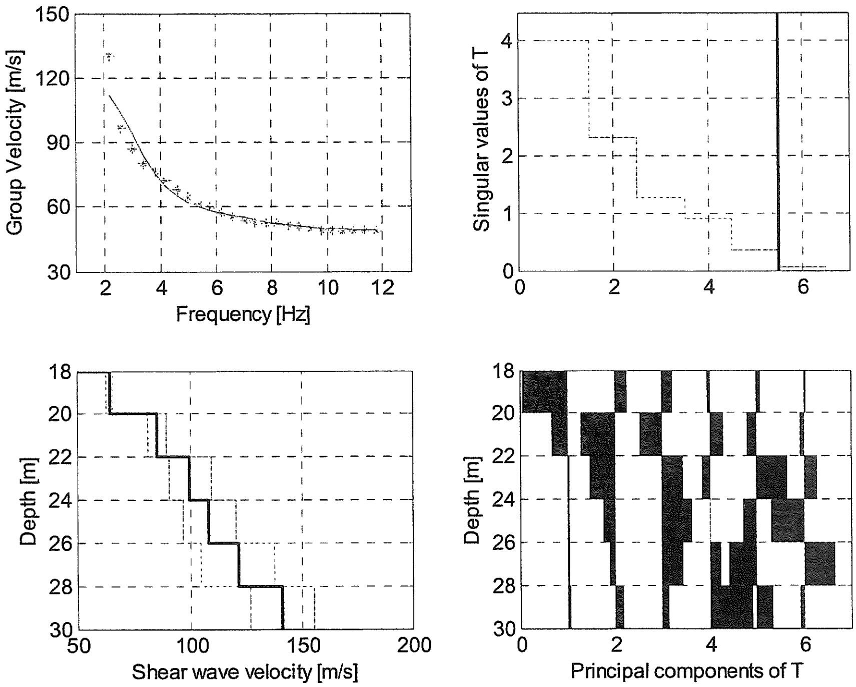

Figure 5. Inversion results of trace-10 in shot P1-86 of Steinbåen site. Top left: Estimated group velocity vs. frequency extracted from the dispersion contour plot (stars) and the model fit as continuous line; Top right: Singular values from SVD and the singular values to the left of the blue thick line are larger than the regularization parameter γ. The corresponding singular vectors, that constitute the resulting shear wave velocity vector, are marked with blue in the bottom right panel; Bottom left: Shear wave velocity vs. depth with blue thick line, and error estimates with red thin line.

Figures 6 and 7 show the same results as in figures 4 and 5 but for trace number 15 from the same shot P1-86. Figure 6 (left) shows the dispersion color map of the trace and one can observe two modes: the fundamental mode and a weaker higher order mode. In the inversion we use only the fundamental mode and its measured dispersion curve is seen in the figure 6 (right). From the plot we see that the interface wave velocity is in the order of 40-110 m/s. The inversion results of both traces are in good agreement with each other as can be observed by comparing figures 5 and 7.

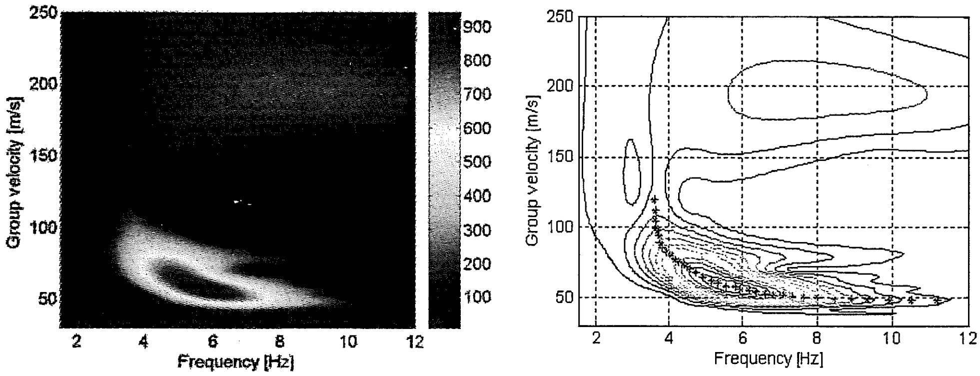

Figure 6. Dispersion analysis same as figure 4 but for trace-15 in the same shot P1-86 in Steinbåen site. Left panel: Dispersion color map of group velocity; Right panel: Same dispersion map but in contour plot and the dispersion curve (stars) is identified by picking the maximum value along the each contour.

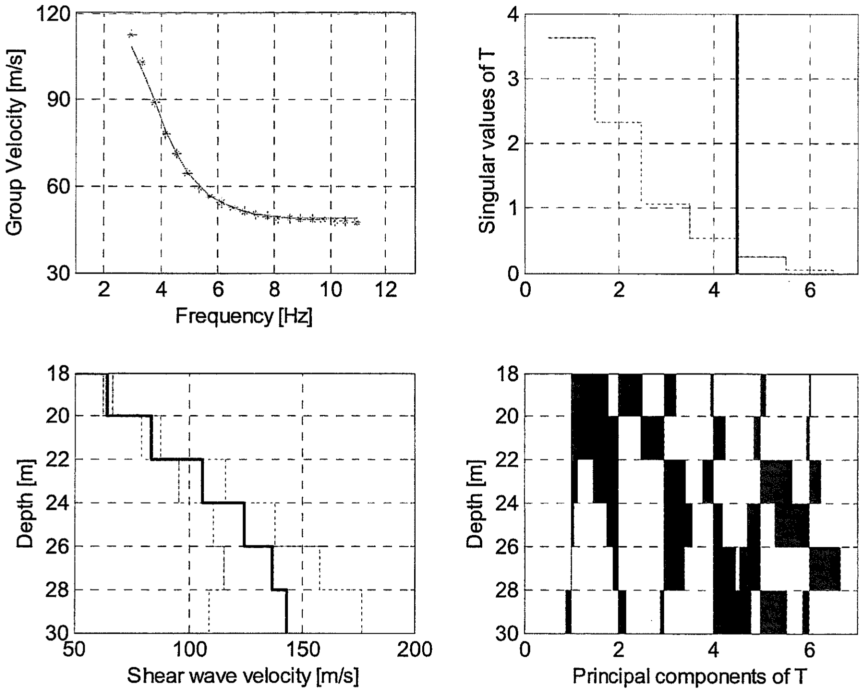

Figure 7. Inversion results same as figure 5 but for trace-15 in the same shot P1-86 of Steinbåen site. Top left: Extracted group velocity vs. frequency from the dispersion contour plot (stars) and the model fit as continuous line; Top right: Singular values from SVD and the singular values to the left of the blue thick line are larger than the regularization parameter γ. The corresponding singular vectors, that constitute the resulting shear wave velocity vector, are marked with blue in the bottom right panel; Bottom left: Shear wave velocity vs. depth with blue thick line, and error estimates with red thin line.

In the multi-trace method the principal components are used to locate the spectral line in the wave number spectra and thereafter the dispersion in frequency-wave number domain is transformed to frequency-phase velocity domain. Figure 8 shows the dispersion analysis of the interface wave data from the same shot P1-86 (the right panel of figure 3). From the figure only one mode is found and the black stars denote the dispersion data and the red circles are the samples of the dispersion curve that are used in the inversion. If one compares this dispersion curve with the one from the WT technique one should remember that the figure 8 represents the phase velocity and not the group velocity obtained by the single-trace method that are shown in figures 4 and 6. The inversion results using the dispersion curve of figure 8 are shown in figure 9.

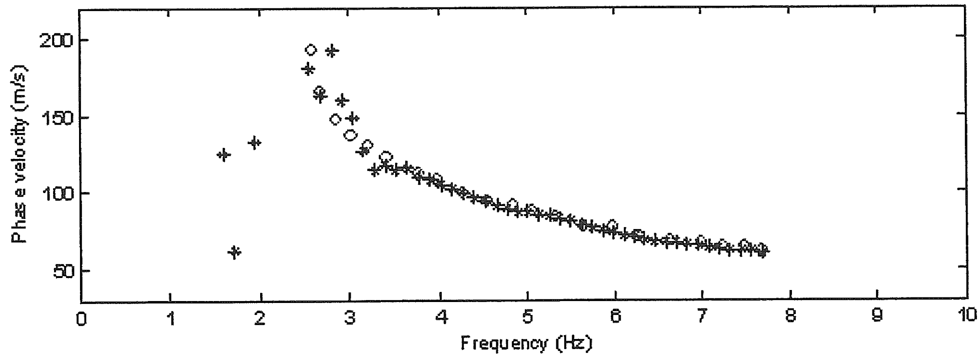

Figure 8. Dispersion analysis of multi-trace data from shot P1-86 in Steinbåen site. Black stars denote the measured dispersion data from the interface waves and the red circles are the sampling of the data used in the inversion.

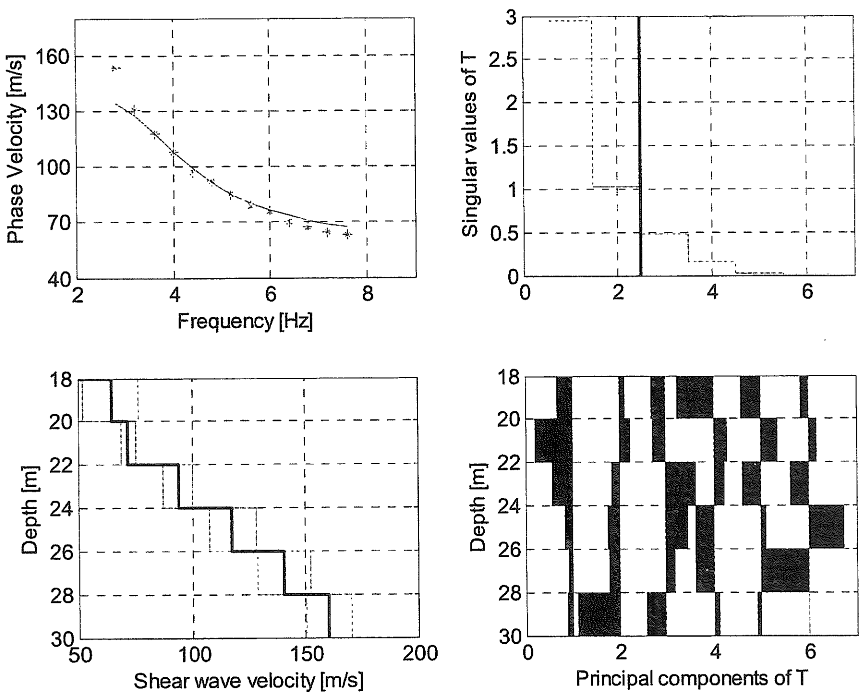

Figure 9. Inversion results of multi-trace analysis of shot P1-86 of Steinbåen site. Top left: Extracted group velocity vs. frequency from the dispersion contour plot (stars) and the model fit as continuous line; Top right: Singular values from SVD and the singular values to the left of the blue thick line are larger than the regularization parameter γ. The corresponding singular vectors, that constitute the resulting shear wave velocity vector, are marked with blue in the bottom right panel; Bottom left: Shear wave velocity vs. depth with blue thick line, and error estimates with red thin line.

In figure 10 the results of the inversion are compared with the measured values from the core samples. The inverted shear wave velocity by multi-trace method is represented by a blue line with squares, the inverted results of trace-10 and trace-15 by single-trace method are plotted in red line with triangles and pink line with circles. The results from the core testing are denoted by black star and diamond. As mention before we had only two values of the shear wave velocity from the core samples, at depths 2.5 and 2.7 meter, but these compared quite well with the inversion results. The difference is in order around 12% compared with the results of the single-sensor method and about 24% compared with the multi-sensor method. In both cases the measured core values are higher than the values of obtained by inversion of the interface waves.

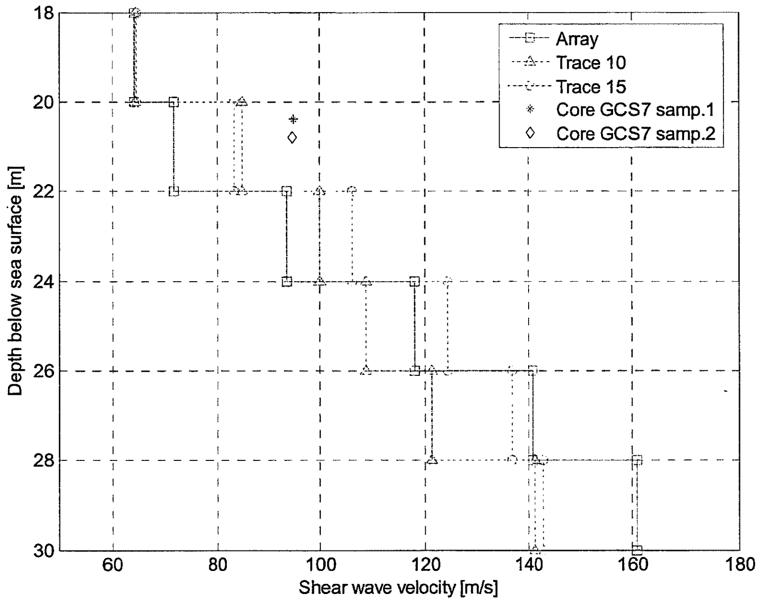

Figure 10. Comparison of the shear wave profiles from analyzing data from P1-86 at Steinbåen and the geotechnical results at the same location. Blue line with squares for multi-trace method; Red line with triangles for trace-10 by single-trace method; Pink line with circles for trace-15 by single-trace method; Black star for geotechnical site testing of core GCS7 sample 1 and black diamond for geotechnical site testing of core GCS7 sample 2.

6 Conclusion

This paper has used different methods to extract the dispersion curve of the interface wave and estimated shear wave velocity profile as function of depth by inverting the dispersion curves. The single-trace method gives the dispersion of the group velocity by using the wavelet transform which has a continuously varying filter bandwidth and provides better velocity-frequency resolution imaging compared with the Gabor analysis. The single-sensor method can also be utilized to study velocity variations with range. The multi-trace method estimates dispersion of phase velocity by principal components method and assumes seabed parameters are range independent beneath the receiving array. The estimated shear wave velocities from the different dispersion curves by using single-sensor and multi-sensor method, respectively, are in good agreement and the comparison with the geotechnical site testing shows a good agreement with an uncertainty of less than 12% in the single-sensor method and less than 24% in the multi-sensor technique. Most important is that the analysis and inversion of recorded interface waves give estimates of shear wave velocity as function of depth in the bottom, in this case down to 10 meters in the sediment.

7 Acknowledgements

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