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Application of the Method of Back Propagation for Acoustic Scattering From an Object in a Waveguide

Michael D. Collins
Naval Research Laboratory
Washington, DC 20375

Abstract: The parabolic equation (PE) method is efficient for solving acoustic scattering problems involving a bounded object in an ocean waveguide. If environmental variations in the proximity of the scatterer are weak, accurate solutions can be obtained by initializing the PE in the farfield with the half-space scattered field. In this paper, the PE scattering model is extended to handle problems involving a scatterer near the ocean bottom or a strong oceanographic feature. The accuracy and efficiency of the model are also improved. An improved PE starting field for the scattered field is constructed by using the two-dimensional PE for the half space to back propagate the half-space scattered field from the farfield to the range of the scatterer.

1. INTRODUCTION

Several computational models have been developed for acoustic scattering from a bounded object submerged in the ocean [1-4]. An approximate solution based on the parabolic equation (PE) method [5,6] achieves an attractive combination of accuracy and efficiency by decoupling the scattering and propagation aspects of the problem [4]. The PE scattering model is a generalization of a matched-asymptotic solution [7] for the field due to a point source [8] and has been implemented for three-dimensional and time-domain problems.

The inner solution is the half-space scattered field due to the incident waveguide field. The inner solution is valid in the inner region, which consists of a vertically-oriented cylinder surrounding the scatterer. The inner solution does not account for reflection from the ocean bottom or refraction due to sound-speed variations in the water column. Beyond the inner region, the scattered field satisfies the PE for the ocean waveguide, with the inner solution as the PE starting field. In general, the three-dimensional PE (3DPE) is required in an intermediate region (which may extend to infinity if there are large three-dimensional variations in the environment). If horizontal coupling [9,10] is not important in the farfield, the two-dimensional PE (2DPE) is valid in the outer region.

The PE scattering model is most efficient for problems involving weak environmental variations near the scatterer because the intermediate solution is not required. If the scattered field is desired in only one direction, it is necessary in this case to compute the scattered field (which is a difficult task) only on a vertical line (rather than on a cylinder) in the desired direction. As environmental variations near the source increase, the inner region becomes smaller. If the inner region does not intersect the outer region, the intermediate solution is required. For some problems, the inner region does not intersect the intermediate region, and the original PE scattering model is not valid.

In this paper, the PE scattering model is improved using the method of back propagation. The half-space scattered field due to the incident waveguide field is back propagated along a radial to the source range using the 2DPE for the half space. The scattered field is obtained by marching the back-propagated field outward with the 2DPE for the ocean waveguide. Numerical results are presented to demonstrate that the back-propagation PE scattering model is accurate for problems that the original PE scattering model can not handle. The back-propagation PE scattering model is more efficient than the original PE scattering model because scattering computations are significantly reduced and 3DPE calculations are not required.

2. THE BACK-PROPAGATION METHOD

For a point source, the homogeneous half-space field may be used to initialize the PE a few wavelengths away from the source [8]. This PE starter is accurate if environmental variations are weak near the source. The half-space PE starter has been generalized to handle larger environmental variations near the source using the method of back propagation [11]. The PE for the half space is used to back propagate the half-space field from the farfield to the source range (for back propagation, the field is marched backwards with the outgoing PE so that phases are reversed). The half-space rays are reversed and collapsed onto the source. The back-propagated field is applied to initialize the PE for the ocean waveguide. The rays emanate from the source with the correct directions, phases, and amplitudes and continue to propagate in the forward direction being reflected and refracted by variations in the environment.

The original implementation of the PE scattering model is analogous to the half-space PE starter for a point source. The homogeneous half-space scattered field due to the incident waveguide field is used to initialize the PE on a cylinder containing the scatterer. In general, the 3DPE is used to march the solution out in range. If three-dimensional variations in the environment are sufficiently gradual, the 2DPE is used when horizontal coupling diminishes in the farfield. This solution is accurate if environmental variations are weak near the scatterer and if the radius of the cylinder is chosen properly.

The cylinder must be sufficiently narrow so that the half-space solution is accurate for the rays that represent energy trapped in the waveguide [8]. The cylinder must be sufficiently wide so that the spreading term may be removed analytically from the wave equation to derive the PE. If possible, the radius of the cylinder is chosen sufficiently large so that the azimuthal radiation pattern of the half-space scattered field is stable. In this case, there is no intermediate region in which the 3DPE is required, and it is necessary to compute the half-space scattered field only in the azimuth of interest.

For some problems involving environmental variations near the scatterer, it is not possible to satisfy both of the requirements on the radius of the cylinder, and the matched-asymptotic solution breaks down. To reduce this limitation, we generalize the PE scattering model with the approach used to improve the half-space PE starter. In the most general implementation of this approach, the half-space scattered field is back propagated to the range of the scatterer using the 3DPE for the homogeneous half space so that the rays are reversed and collapsed through the surface of the scatterer. The back-propagated field is used to initialize the 3DPE for the ocean waveguide. The rays pass back through the surface of the scatterer with the appropriate directions, phases, and amplitudes and an accurate solution is obtained in the farfield.

3. EFFICIENT IMPLEMENTATION

The back-propagation PE scattering model is valid for problems that the original PE scattering model can not handle. However, the implementation described in Section 2 is inefficient because it is necessary to compute the half-space scattered field in all directions and use the 3DPE even if the scattered field is desired in only one direction. In this section, we describe an efficient implementation of the back-propagation PE scattering model. We assume that horizontal coupling is negligible in the farfield.

Since rays from different locations on the surface of the scatterer are essentially confined to a vertical two-dimensional plane in the farfield, the 2DPE for the half-space is used to back propagate the half-space scattered field in a vertical plane. The back-propagated field is forward propagated with the 2DPE for the ocean waveguide. Although the 2DPE does not back propagate the rays to the correct locations on the surface of the scatterer, they arrive at the correct locations in the farfield when they are subsequently forward propagated with the 2DPE. This solution is valid in the farfield for problems that the original PE scattering model can not handle. If the scattered field is desired for only one azimuth, efficiency is improved significantly because the half-space scattered field (which is difficult to compute) is required on a line rather than a cylinder and the 3DPE is not required.

To test the accuracy of the back-propagation PE scattering model, we consider a problem involving an array of sources in a range-independent ocean. This type of problem is easily solved and is essentially identical to the scattering problem if the array is constructed properly. Three 50-Hz sources are placed in phase at a depth of 180 m in an ocean of depth 200 m. The (x,y) positions of the sources in meters are (50,0), (-50,0), and (10,20). The sound speed is 1500 m/s in the ocean and 1700 m/s in the sediment. The density is 1.5 g/cm^3 and the attenuation is 0.5 dB/λ in the sediment.

To obtain an accurate reference solution for this problem, we apply the PE for each of the point sources and superimpose the three acoustic fields. Since the sources are close to the ocean bottom and separated by 100 m, it follows from the asymptotic results of [8] involving the critical angle at the ocean bottom that the original matched-asymptotic solution is not valid for this problem even if the 3DPE is used. The matched-asymptotic solution is generated for this problem using the half-space field to initialize the 2DPE in the direction of the positive x-axis. The solutions appearing in Figure 1 for three locations of the matching range all have large errors.

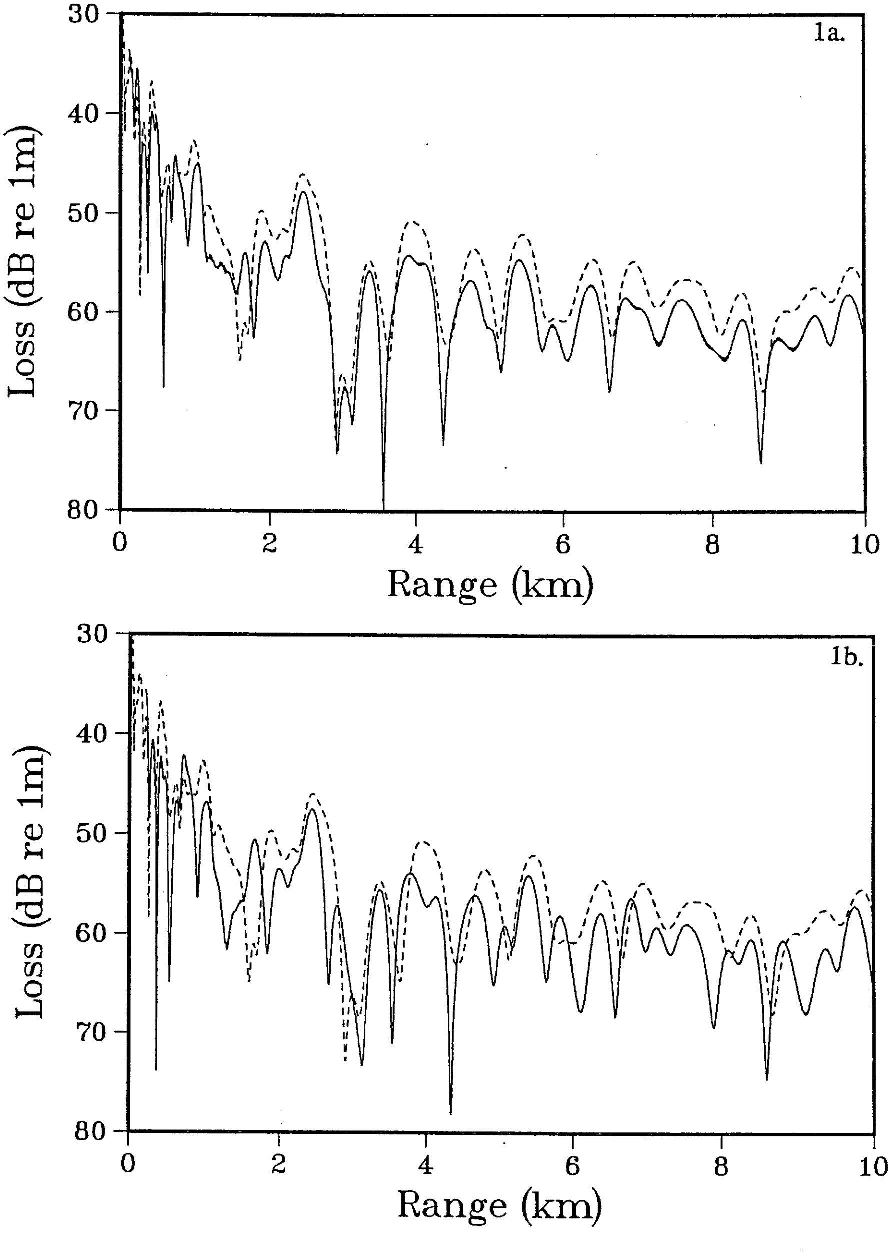


Fig. 1. Transmission loss at a depth of 100 m for a problem involving three point sources. The dashed curves are the reference solution.

The solid curves are the matched-asymptotic solution involving the half-space field and the 2DPE without an intermediate region. The half-space field is used to initialize the 2DPE at the ranges of: (a) 100 m, (b) 200 m, and (c) 500 m.

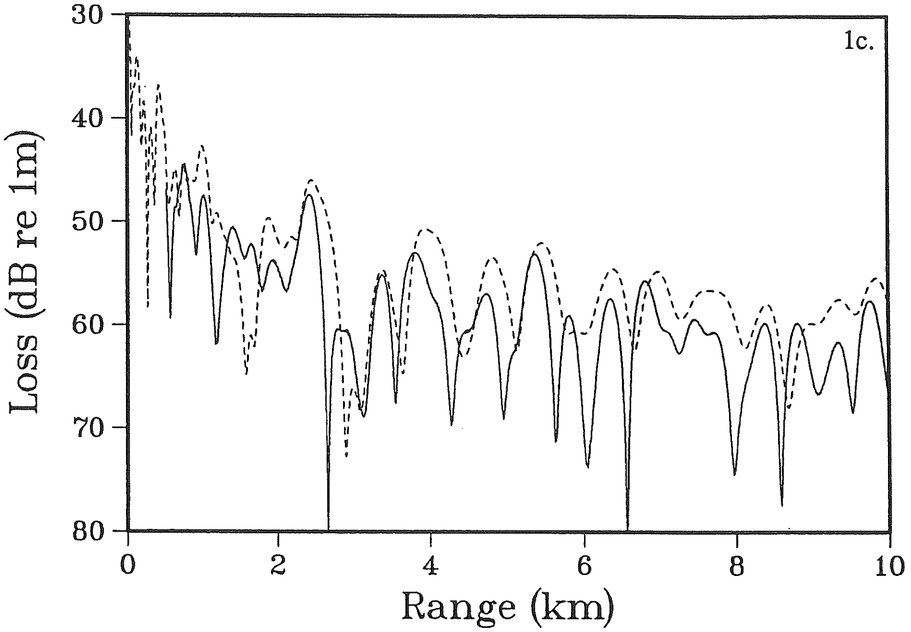
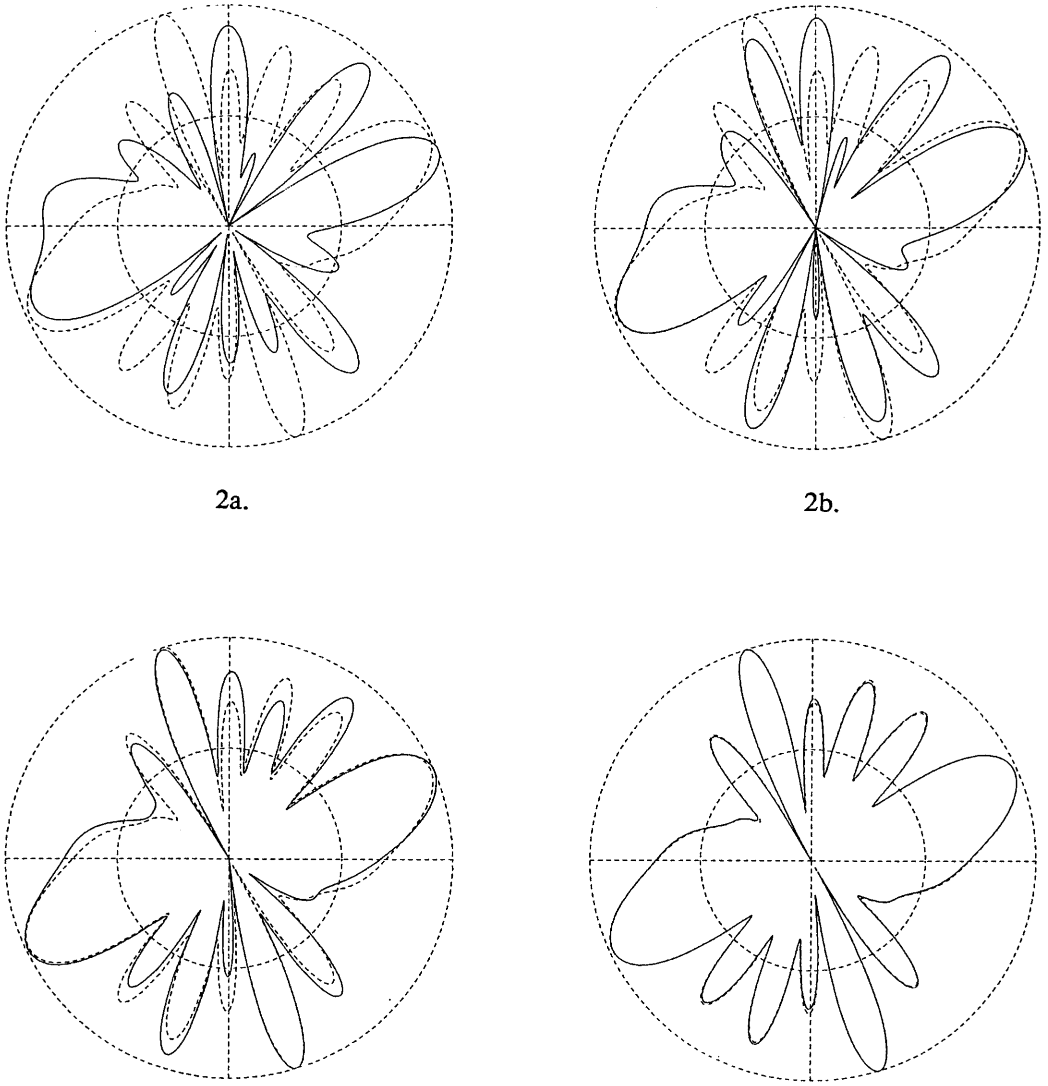


Fig. 1 (cont'd). Transmission loss at a depth of 100 m for a problem involving three point sources. The dashed curves are the reference solution.

The radiation pattern of the free-space field at the source depth appears in Figure 2 for various ranges. The radiation patterns differ significantly from the farfield pattern at 100 and 200 m. The radiation pattern begins to settle down at 500 m and is nearly stable at 3 km. We compute the half-space field at a range of 3 km and back propagate the field to the range origin using the 2DPE for the half space. Appearing in Figure 3 is the field forward propagated with the 2DPE for the ocean waveguide. The back-propagated PE scattering solution is in excellent agreement with the reference solution in the farfield. As one would expect, there is some error prior to about 500 m in range, where the radiation pattern begins to settle down.

4. CONCLUSION

The PE scattering model has been improved using the method of back propagation. The back-propagation PE scattering model is accurate for problems that the original PE scattering model can not handle. The improved model is also more efficient for some problems because the intermediate region is not required. Thus it is only necessary to compute the half-space scattered field on a line rather than a cylinder and it is not necessary to use the 3DPE. The accuracy of the back-propagation PE scattering model was demonstrated for a problem involving an array of point sources.



2a.

2b.

2c.

2d.

Fig. 2. Radiation patterns (in decibels) on a circle at the depth of the sources for a problem involving three sources. The dashed curves are the farfield radiation patterns. The solid curves are the radiation patterns at the ranges of: (a) 100 m, (b) 200 m, (c) 500 m, and (d) 3 km. The radius of the outer circle is 16 dB.

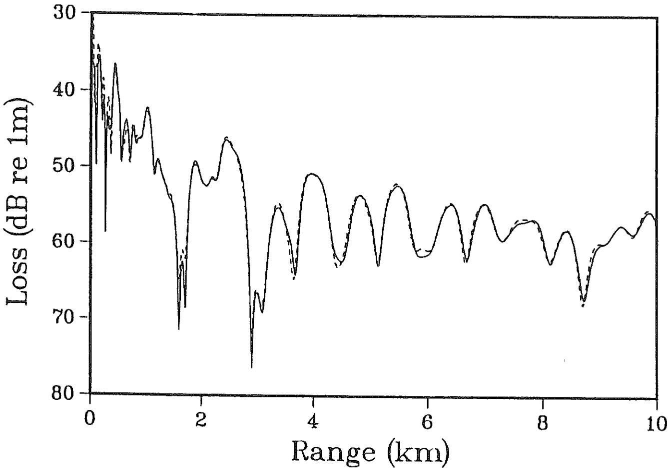


Fig 3. Transmission loss at a depth of 100 m for a problem involving three point sources. The dashed curve is the reference solution. The solid curve is the solution generated by back propagating the half-space field with the 2DPE for the half space and forward propagating with the 2DPE for the ocean waveguide.

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